

How to Judge Voting Schemes

Amartya Sen

In his book *In Westminster Abbey*, John Betjeman describes the things “our Nation stands for.” His list includes “democracy and proper drains.” Both are, of course, very important, but there is much more agreement on the requirements of a good drainage system than on the specifications of democracy. One of the areas of disagreement is the choice of voting schemes. There are plenty of alternative schemes that have been used in systems with democratic credentials. A great many other schemes have been proposed, defended, attacked, and shelved. The subject remains of intense interest, not least because of the importance of finding “institutional structures that can improve the chances for the sustenance and spread of representative democracy in the world,” as Roger Myerson puts it in this issue. The symposium includes a number of interesting contributions in this important area of research.

Overview of the Symposium

The collection begins with a most helpful introductory essay by Levin and Nalebuff on different “vote-counting schemes.” The authors explain what these alternative schemes are, how they work, to what extent they differ, and how they might influence the voter’s choice of strategy. There is also an interesting empirical claim in the Levin-Nalebuff paper. Even though these different voting schemes *can* give very different results, depending on the nature of the individual preferences and their similarity and mismatch, Levin and Nalebuff

■ *Amartya Sen is Lamont University Professor, Harvard University, Cambridge, Massachusetts. During 1994, he was President of the American Economic Association.*

use data from elections held in British organizations to argue that whatever voting method had been chosen, it would actually have yielded the same or very similar outcomes. This raises the question whether the choice between these different voting schemes should be less agonizing than their formal differences suggest.¹

Of the other five papers, three are concerned with specific voting schemes. Nicolaus Tideman and Robert Weber discuss and illuminatingly evaluate two particular classes of voting schemes, “single transferable vote” and “approval voting,” considering variations within each class and proposing fresh variants. Based on their analyses of the outcomes and of the nature of the processes used, Tideman and Weber identify good reasons for viewing these approaches favorably. The evaluative methods used concern the working of the voting systems as a whole, rather than their ability to satisfy prespecified axiomatic properties, as in classical social choice theory that takes off from Arrow’s (1951) axiomatic procedure. I shall come back to this question later.

Peyton Young explores the requirements of “optimum group decisions,” nicely contrasting Condorcet’s approach with Borda’s (on which more presently). He pays particular attention to the “maximum likelihood method,” explored by him in Young (1988), as a procedure much in the spirit of Condorcet’s general approach.²

Douglas Rae’s paper discusses a specific problem of immediate practical importance, to wit, that of minority representation when elections are based on single-member constituencies.³ In the extreme case, when a minority in the nation is a minority in every constituency, it can go without representation altogether. One way of dealing with this problem of minority representation that is favored in many countries (while sticking to single-member constituencies) is through redefining the constituencies in such a way as to help (or even ensure) the victory of a certain number of minority candidates. Rae points out that this procedure, which is really gerrymandering for the sake of minority representation, creates its own problems: in particular, it gives the state too large a role in deciding “which minorities, at what locations” will be favored.⁴

¹Levin and Nalebuff note that these findings “suggest a connection” with an invariance result presented by Caplin and Nalebuff (1988, 1991). This is so in a very general sense, but it must be emphasized that neither set of regularities is, in any way, a corollary of the other. Each indicates an invariance of outcomes in a family of voting procedures—with different families in the two exercises. Caplin and Nalebuff’s important analytical results, which have aroused much interest, use explicitly postulated similarities in voters’ preferences, whereas Levin and Nalebuff’s striking empirical findings in this journal are influenced by the actual patterns of votes in some British elections. The connection, which is yet to be established, would be worth exploring.

²Young (1988) also has antecedent connections with Kemeny’s (1959) solution involving minimal pairwise violations of individual preferences. See also Levin and Nalebuff’s discussion of the “Kemeny-Young method,” and also of the “Jech method” and the “Kendall-Wei method.”

³Rae’s analysis relates to issues that recently received much public attention due to Lani Guinier’s (1994) work.

⁴On related matters, see also Anyang’Nyong’o (1994), who discusses the limitations of single-member constituencies particularly when the government has an authoritarian background. He illustrates the difficulties with an analysis of the Kenyan general elections of 1992.

Rae argues for multimember constituencies over larger districts, which gives greater chance to democratic representation of minorities.

Rae's argument is quite persuasive. But there must be some give-and-take here, particularly in deciding on *how large* the constituencies should be. Proportional representation for the country as a whole would be the other extreme case—opposite to that of single-member constituencies—giving extensive scope to the national electorate to choose minority representation without arbitrary constraints imposed by the government. Presumably there would be some kind of a reasonable division that balances 1) the importance of local knowledge and personal involvement, which favors small constituencies, and 2) the value of unfettered choice in minority representation for the country as a whole, which favors large constituencies. Given the force of Rae's critique of single-member constituencies, this further issue is all the more important.

In an insightful general paper, Myerson discusses the use of techniques and results obtained in economic theory to enlighten political modelling. The possibility of using "spatial" models, with specified metrics that identify "distances" between alternative outcomes, is a good point of connection. Myerson also draws attention to the trickiness in translating results and intuitions from one field to another, since the substantive exercises can be quite different despite formal similarities. For example, close emulation of a rival's position may be useful both in the economic exercise of "placing" a new commodity (or locating a shop) and in the political exercise of choosing a platform for an election. But in the standard economic exercise both commodities (or shops) will get established, whereas in the typical political exercise only one alternative candidate *or* the other will actually get chosen.

In addition to individual results and insights to be found in these papers, the symposium suggests some interesting general issues. One concerns the choice of "inputs" into the voting schemes, on which there is remarkable uniformity among the different proposals considered here and the way their workings are interpreted. The canonical form is taken to be each voter's given ranking of all alternative candidates or informational structures derived from such rankings. A second general issue concerns the method of evaluation or judgment of the respective voting schemes, which departs from the standard axiomatic methods used in social choice theory. I shall discuss both questions in turn.

Voting as a Process and Menu Dependence

As far as the choice of *inputs* is concerned, all the voting schemes considered here explicitly use, or draw on, the ranking of each voter of *all* the alternatives considered *together*. In some cases the voting input is directly this entire ranking, as with the so-called "Borda method," "single transferable vote," or "Coombs voting." In other cases, as with "plurality voting," the inputs

are single votes for a particular candidate from a list. Sometimes the inputs are paired comparisons, as with “Copeland voting.”⁵ But all of these “inputs” are taken to be derived from a full ranking that each voter has over *all* the alternatives, and this is assumed (implicitly) to be invariant with respect to the choices offered—that is, independent of the “opportunity set” (the “menu”) from which the voter chooses. In the analyses presented, there is never a need to go beyond each voter’s one basic ranking over all candidates.

This, it must be noted, is not just a matter of description of the voting schemes themselves, even though the schemes limit the possibilities of expression in particular ways. In interpreting a vote over a group of candidates (say, in a plurality voting) or an expressed full rank ordering of a particular “menu” of candidates (say, in a Borda procedure), it is not necessary to assume that each voter has a *menu-independent* preference ordering, even though the comparative exercises performed in this symposium (for example, by Levin and Nalebuff, or by Young) proceed on that implicit presumption. The observation of a particular act of voting under any of these schemes does not tell us whether the rankings are dependent on the “menu” or not. This is an additional assumption. If the voter were offered a choice over a *subset* of the set over which that voter has expressed a ranking, we *may or may not* presume that the voter would stick to the same overall ranking applied to this particular subset (or, to put it formally, would simply express the “restriction” of the fuller ranking over that subset).

In fact, if the *process* of voting is taken seriously, there can be good reasons for such a menu-dependence. The act of voting for x can be seen as the act of voting “for x from set S ” (let us denote it x/S). For example, the presence of a “green” candidate z may make a voter go for a somewhat “greenish” x over environmentally naive but otherwise sensible candidate y , even though she might have voted for y over x had there been no fully “green” candidate. This kind of “non-binariness” can arise from several different reasons. For example, the presence of a “green” candidate in an election can make the voter decide that the environment is likely to be an active issue in postelection politics (thereby giving an edge to x over y). Or the voter might find it “distasteful” to vote *against* a green candidate (z) in favor of someone (y) completely innocent of the environment (even though she may not wish to go so far as to vote for the very green z). Neither thought need prevent the voter from voting for y over x , if they are the only two candidates; the presence of the third candidate changes, in this case, the ranking of x and y .⁶

Menu-dependent choice behavior can arise from other reasons as well, as I have tried to discuss in Sen (1993). Formally, this indicates that the choice

⁵All of these methods are presented in the Levin and Nalebuff paper in this issue, although they use a somewhat different principle of classification of voting schemes.

⁶I should emphasize that this is a different issue from the one involved in Arrow’s (1951) condition of “independence of irrelevant alternatives.” The problem here concerns the nature of individual preferences or choices, not the relation between individual preferences and social choice (as in Arrow’s condition). On this, see Sen (1970, 1986).

function is not “binary,” that is, not fully representable by a given binary relation over the set of all alternatives. There is some obvious advantage in having voting schemes that take the more general form of operators that transform the set of choice functions (rather than given binary relations) of all the individuals in a group into choice functions for the group as a whole (social or collective choice functions).⁷ The choice functions may or may not be binary. On the other hand, using choice functions as inputs is much more demanding on voters and on the system of “counting” than is the use of rankings of each voter with menu-dependence assumed away. In fact, using choice functions as inputs may be infeasible in practice in many types of exercises. Levin and Nalebuff may, thus, have good reasons to confine their investigation to the schemes that operate on individual rankings or simple choices.

This does not, however, settle the issue of *interpretation* of voter preference on the basis of observed choice (even in the absence of considerations of strategic manipulation). If a person votes for x in a choice over (x, y, z) , this need not tell us that such a voter would prefer x over y in a two-way contest, since preferring $x/(x, y)$ over $y/(x, y)$ is not the same as preferring $x/(x, y, z)$ over $y/(x, y, z)$. Similarly, the *ranking* of (x, y, z) need not tell us how (x, y) will be ranked. These differences can alter the relationship between the different voting schemes as presented in many of these papers, which assume menu-independence of individual choices and rankings.

The presumption of menu-independence is common enough, and to invoke it is no great crime, but it needs to be stated. It is also worth noting that this assumption has the effect of reducing the importance of the *process* of voting, including seeing an act of voting as a vote *for* someone *against* other candidates and treating the choices offered as having epistemic value in determining the nature of the elections in which one is taking part. The relationships between the different voting schemes, as discussed here, involve that implicit assumption.⁸

Axiomatic Analysis and Synthetic Properties

In their overview, Levin and Nalebuff do not enter the business of finding the “best” voting scheme. But insofar as the different voting schemes are evaluated, as they are in passing, this is done *not* in terms of the classical social choice approach, initiated by Arrow (1951), of first postulating a set of axioms

⁷Fuad Aleskerov (1994) has classified and contrasted different categories of voting models, in particular three classes of operators. All the schemes considered in this symposium fall, directly or indirectly, into only one of the three categories explored by Aleskerov: “operators which transform individual binary relations into collective choice functions.” The use of menu-dependent preferences as inputs can be handled by general operators that “deal with individual and collective choice functions.” See also Aizerman (1985) and Aizerman and Aleskerov (1986), and on related matters, also Aizerman and Malishevski (1981).

⁸This proviso would apply, to a great extent, also to the important analytical results contained in Caplin and Nalebuff (1988, 1991).

demanding specific properties and then checking which of these properties the respective voting schemes satisfy. Rather, what they do is discuss the motivation behind each proposal, how they actually operate, and the kinds of results they yield. They discuss, in this context, some of the noteworthy operational properties they have: characteristics of each proposal *taken as a whole* such as legitimacy, representativeness, and ability to encourage participation or to discourage the formation of many political parties.

This raises a methodological issue concerning our ability to judge alternative procedures in terms of how they function as a whole, rather than in terms of isolated properties used as axioms. This consolidated rather than axiomatically analytical approach to evaluation is also used in the other papers, including those by Rae, Tideman, Weber, and Young.⁹

The case for Arrow-like axiomatic use of isolated properties to assess political systems rests mainly on the belief that a synthetic view hides many things, and these could be captured by prespecifying the requirements of good functioning and then checking whether they are or are not satisfied by a particular voting scheme. There is clearly much merit in that line of reasoning. On the other hand, isolated properties may also be very difficult to judge on their own, as we know from the literature on social choice theory itself, initiated by Arrow. We may like each of a set of axiomatic requirements, but as Arrow's "impossibility theorem" and related results show, a set of reasonable-looking conditions can together yield an impossibility, and then obviously we cannot continue to insist on each. More generally, we have learned from social choice theory that the "rub" of an isolated property does depend on the other requirements with which it is combined (Sen, 1970, 1986).

The isolated-property line of reasoning was explicitly introduced by Condorcet (1785), demanding that an alternative should be chosen if and only if it beats every other alternative in *pairwise* contests. This is certainly an appealing rule, and one that Borda's procedure of rank-order voting fails to satisfy (an issue that Young discusses well).¹⁰ But the reason why Borda fails is precisely because it takes note also of the position of each of the *other* alternatives, which may not be irrelevant in deciding which one to choose from a given set of options. For example, if a million people have the ranking (in descending

⁹Peyton Young does, however, end with presenting a brief and neat axiomatic justification of the maximum likelihood rule; this takes the form of identifying the properties that this rule has and that define it uniquely enough to make it the only rule satisfying these properties. The properties in question are the well-known qualities of anonymity, neutrality, and the Pareto principle, and two freshly defined characteristics called "reinforcement" and "local independence of irrelevant alternatives." This is a use of the Arrow-style axiomatic method in full force, but even for Young, the case for the rule under examination does not rest only—or even primarily—on this line of justification, and much attention is paid to the way the rule functions *as a whole*.

¹⁰Young suggests that any positional scoring rule would have certain properties that generalize Borda's particular method. Young is right. In fact, this class of generalization has been extensively—and fairly exhaustively—investigated by Gardenfors (1973) and Fine and Fine (1970). The particular regularity that Young notes does, in fact, follow from these results.

order) x, y, z , and a million minus one people order y, z, x , then there is indeed a case for selecting y , who is liked best or second best by all, rather than choosing x , who is put last by virtually half the people. And yet the clear “Condorcet winner” is x . If isolated properties provide illumination, so do the overall workings of voting procedures.

It is, nevertheless, true that the merits and demerits of a voting scheme, judged as a whole, can be put, eventually, into an axiomatic framework. The previous example is no exception to this. But the process of illumination is not necessarily best served by sticking exclusively to the postulation of isolated properties identified first. This is something of a defense of the kind of procedure that is used in many of the papers in this collection, in contrast with the classical social choice procedures. On the other hand, this need not be seen to be an argument against the usefulness of the axiomatic approach. It is rather a question of how the appropriate axioms be considered and chosen. If a voting scheme seems to yield a result that is eminently criticizable, it is appropriate to ask *why* is it criticizable, and that exercise of isolation will yield a property that is violated and which can then be used—at least tentatively—as an axiomatic requirement against which other procedures may be judged.

The exercise of evaluation of voting schemes is, thus, best seen as a two-way process, part of it involving the classical social choice approach of going from axiomatic demands of isolated properties to whole voting schemes, and another part dealing with examining the attractions and perversities of voting schemes and isolating the properties responsible for these characteristics.¹¹ Our values need not take *only* the a priori analytical route, and the approach of judging the workings of fully specified schemes, much used in this symposium, can be seen as complementary rather than competitive with the traditional social choice procedures.

■ *For helpful comments, I am grateful to Emma Rothschild, and, for research support, to the National Science Foundation.*

¹¹Something similar can be said, incidentally, about other types of selection problems in which axiomatic methods are used. For example, in assessing the approach of “expected utility,” compared with its rivals, there may also be a good case for seeing how they respectively function as a whole, rather than beginning with isolated properties (such as completeness, continuity, independence, and so on). After all, weighing alternatives by their likelihood has some appeal of its own, which may be no less intuitive than the particular isolated requirements used in the standard axiomatic derivations of expected utility.

References

- Aizerman, M. A.**, "New Problems in the General Choice Theory," *Social Choice and Welfare*, December 1985, 2, 235–82.
- Aizerman, M. A., and F. Aleskerov**, "Voting Operators in the Space of Choice Functions," *Mathematical Social Sciences*, June 1986, 11, 201–42.
- Aizerman, M. A., and A. V. Malishevski**, "General Theory of Best Variants Choice: Some Aspects," *IEEE Transactions on Automatic Control*, 1981, AC-26, 1031–41.
- Aleskerov, Fuad**, "Voting Models in the Arrowian Framework." In Arrow, K. J., et al, *Social Choice*. Proceedings of the International Economic Association roundtable conference at Hernstein, Institute of Control Sciences, Moscow, forthcoming 1994.
- Anyang'Nyong'o, P.**, "The Pitfalls of Constituency Representation: The Case of Kenya," paper presented at the Nobel Symposium on "Democracy's Victory and Crisis," Uppsala University, August 27–30, 1994.
- Arrow, Kenneth J.**, *Social Choice and Individual Values*. New York: Wiley, 1951, 2nd edition, 1963.
- Caplin, Andrew, and Barry Nalebuff**, "On 64%-Majority Rule," *Econometrica*, July 1988, 56, 787–814.
- Caplin, Andrew, and Barry Nalebuff**, "Aggregation and Social Choice: A Mean Voter Theorem," *Econometrica*, January 1991, 59, 1–24.
- Condorcet, Marquis de**, *Essai sur l'Application de l'Analyse a la Probabilite des Decisions rendues a la Pluralite des Voix*. Paris: L'Imprimerie Royale, 1785.
- Fine, Ben, and Kit Fine**, "Social Choice and Individual Ranking," *Review of Economic Studies*, July 1974, 41, 303–22.
- Fishburn, Peter C.**, *The Theory of Social Choice*. Princeton: Princeton University Press, 1973.
- Gardenfors, Peter**, "Positional Voting Functions," *Theory and Decision*, September 1973, 4, 1–24.
- Guinier, Lani**, *The Tyranny of the Majority: Fundamental Fairness in Representative Democracy*. New York: The Free Press, 1994.
- Kemeny, John**, "Mathematics without Numbers," *Daedalus*, Fall 1959, 88, 571–91.
- Sen, Amartya K.**, *Collective Choice and Social Welfare*. San Francisco: Holden-Day, 1970; and Amsterdam: North-Holland, 1979.
- Sen, Amartya K.**, "Social Choice Theory." In Arrow, K. J., and M. Intriligator, eds., *Handbook of Mathematical Economics*. Amsterdam: North-Holland, 1986, pp. 1073–181.
- Sen, Amartya K.**, "Internal Consistency of Choice," *Econometrica*, May 1993, 61, 495–521.
- Young, H. Peyton**, "Condorcet's Theory of Voting," *American Political Science Review*, December 1988, 82, 1231–44.

This article has been cited by:

1. Sacha Bourgeois-Gironde, João V. Ferreira. 2024. The expressive power of voting rules. *Social Choice and Welfare* **62**:2, 233-273. [[Crossref](#)]
2. Sacha Bourgeois-Gironde, João V. Ferreira. 2022. The Expressive Power of Voting Rules: Theory and Experimental Evidence. *SSRN Electronic Journal* **39**. . [[Crossref](#)]
3. Salvatore Barbaro. 2021. On the dispensability of run-off elections. *Zeitschrift für Politikwissenschaft* **31**:1, 71-82. [[Crossref](#)]
4. Ivan Boldyrev. 2020. Realities of Formalization. *History of Political Economy* **52**:S1, 270-293. [[Crossref](#)]
5. William V. Gehrlein, Dominique Lepelley. Condorcet Efficiency and Social Homogeneity 157-198. [[Crossref](#)]
6. Paulo Trigo Pereira, João Andrade e Silva. 2009. Citizens' freedom to choose representatives: Ballot structure, proportionality and “fragmented” parliaments. *Electoral Studies* **28**:1, 101-110. [[Crossref](#)]
7. Joachim Weimann. DELEGATIONSPROBLEME IN REPRÄSENTATIVEN DEMOKRATIEN 439-451. [[Crossref](#)]
8. Ioannis A. Kaskarelis. 2009. Free market economy, representative democracy, cultural globalisation and the protection of the environment. *International Journal of Environment and Sustainable Development* **8**:2, 132. [[Crossref](#)]
9. Philip N. Howard. 2008. In the Margins: Recounts, Victory and Error in the 2000 and 2004 U.S. Elections. *SSRN Electronic Journal* **23**. . [[Crossref](#)]
10. Joachim Weimann. Kollektive Entscheidungen 155-228. [[Crossref](#)]
11. Joachim Weimann. Delegations-probleme in Repräsentativen Demokratien 409-436. [[Crossref](#)]
12. Steven Pressman, Gale Summerfield. 2000. The Economic Contributions of Amartya Sen. *Review of Political Economy* **12**:1, 89-113. [[Crossref](#)]
13. Joachim Weimann. Kollektive Entscheidungen 150-228. [[Crossref](#)]
14. Shaun Bowler. 1996. Reasoning voters, voter behaviour and institutions: The decision dependence of voting behaviour. *British Elections and Parties Yearbook* **6**:1, 106-127. [[Crossref](#)]