Grading Standards and Education Quality

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We consider school competition in a Bayesian persuasion framework. Schools compete to place graduates by investing in education quality and by choosing grading policies. In equilibrium, schools strategically adopt grading policies that do not perfectly reveal graduate ability to evaluators. We compare outcomes when schools grade strategically to outcomes when evaluators perfectly observe graduate ability. With strategic grading, grades are less informative, and evaluators rely less on grades and more on a school’s quality when assessing graduates. Consequently, under strategic grading, schools have greater incentive to invest in quality, and this can improve evaluator welfare. (JEL D82, I21, I23)

In a 2003 article in the Washington Post, Duke University professor Stuart Rojstaczer summarizes a clear trend in grading at American colleges and universities: “I don’t give C’s anymore, and neither do most of my colleagues. And I can easily imagine a time when I’ll say the same thing about B’s” (Rojstaczer 2003). The trend toward higher grades is often viewed as the result of deteriorating grading standards, which allow students to receive higher grades for lower quality work. While this trend is pervasive, it is not new. When Harvard University considered similar issues in 1894, the committee concluded:

Grades A and B are sometimes given too readily—Grade A for work of no very high merit, and Grade B for work not far above mediocrity. … One of the chief obstacles to raising the standards of the degree is the readiness with which insincere students gain passable grades by sham work.

There are two main concerns about the deterioration of grading standards. First, when high grades are assigned liberally, they convey less information to employers,
graduate schools, and other evaluators about a student’s ability. This distorts hiring and postgraduate admissions decisions, and reduces the likelihood that evaluators select the highest ability candidates. Second, weak grading standards could compromise education quality, allowing schools to use strategic grade assignment—grade inflation, for example—to manipulate evaluators’ perceptions of their graduates, as a substitute for actual investment in developing student ability. Similarly, students may devote less effort to their studies, expecting easy grading.

Our analysis considers the interaction between the strength of grading standards and investments in education quality. We compare a setting of strong grading standards, where grades (or other assessments) fully reveal graduate ability, to a setting of weaker standards, which arise when schools strategically design grading policies as they compete to place students. The model of competition between the schools proceeds in two stages. In the first stage, each school undertakes a costly investment to improve education quality, increasing the likely ability of a graduate.\(^3\) In the second stage, schools simultaneously design grading policies, which determine how transcripts are assigned to graduates of different abilities. Once grading policies are chosen, each school produces a graduate. Next, an evaluator observes investments, school grading policies, and graduate transcripts, updating her beliefs about each graduate’s ability. She then selects a graduate from one of the schools to receive a prize (e.g., job, graduate school admissions, fellowship). The evaluator prefers high-ability graduates, and therefore assigns the prize to the candidate with the highest expected ability. A school benefits when its graduate is selected, regardless of ability. We derive the equilibrium of this game. We then solve the game again, under the alternative assumption that grades (or other post graduate assessments) fully reveal graduate ability to the evaluator; this is the fully informative grading benchmark.

In the equilibrium with strategic grading, schools assign grades that convey some information about graduate ability without perfectly revealing it. Consistent with conventional wisdom and popular discussion, the grading policy implemented by the higher quality school is less informative than the grading policy of the lower quality school (e.g., a higher quality school engages in more grade inflation). Holding school investments fixed, we find that allowing strategic grading makes it less likely that an evaluator selects a high-ability candidate. With *endogenous* investments in education quality, however, allowing strategic grading often has the opposite effect. Giving schools the freedom to strategically choose grading policies (compared to maintaining strict grading standards) changes the incentives for schools to invest in developing student ability. Because strategic grading policies decrease transcript informativeness, the evaluator relies more on a school’s reputation for quality when assessing its students. Since investment in school quality plays a larger role

\(^3\) Rivkin, Hanushek, and Kain (2005) demonstrate that teachers have powerful effects on student achievement and test scores. At the same time, easily observable teacher characteristics like education and experience explain very little of the variation in teacher quality. Thus, identifying, recruiting, and retaining high-quality teachers is a costly investment, with the potential to significantly improve student outcomes. Other types of investment include caps on class sizes, providing opportunities for teacher professional development, increasing the allocation of school resources, and monitoring (and incentivizing) student effort. In Section VI, we present results that include student effort as an essential but unobservable input into the educational process.
in evaluator assessments, schools respond by investing more. Thus, the absence of strict grading standards introduces countervailing effects for evaluators: transcripts are less informative, hindering the ex post selection of high-ability graduates, but schools invest more in education quality, so graduates are more likely to be high ability ex ante. We find that the effect on investment may dominate: despite being less able to distinguish high and low-ability students, the probability that an evaluator selects a high-ability graduate may increase when schools grade strategically.

We then incorporate student effort into the model, treating it as an essential but unobservable input into the production of graduate ability. If school investments are held fixed, then strategic grading can magnify the moral hazard facing students and can induce students to shirk. This is not necessarily the case, however, when we account for endogenous school investments. Our analysis exhibits a strategic complementarity between school investment and student effort: if a student anticipates greater investment by schools, his incentive to shirk decreases. Because schools invest more with strategic grading, in equilibrium the complementarity can dominate students’ moral hazard, leading to a net increase in student effort.

Therefore, the central concerns over weakening grading standards may not be valid once incentives for schools to invest in the quality of education are taken into account. Although the freedom to grade strategically allows schools to distort the informational content of grades (which they do in equilibrium), it also creates additional incentives for schools to improve the quality of education. Compared to a setting in which grades are necessarily fully informative, the increase in equilibrium education quality can lead employers, graduate schools and other evaluators to be better off. Additionally, because of strategic complementarities between student effort and educational investment, strategic grading may increase student effort.

The next section surveys related literature. Section II describes the model. Section III analyzes the model with strategic grading. Section IV considers a benchmark with strong evaluation standards, in which grading is fully informative. Section V compares outcomes of the strategic grading game with the full information benchmark, establishing our main results. Section VI incorporates unobserved student effort into the analysis, extending our results. Section VII discusses some of the assumptions of the model in more detail. Section VIII concludes with a brief discussion of alternative interpretations of the model and summarizes its main implications. Proofs of all results are in the Appendix.

I. Related Literature

A significant portion of the economics literature on grade assignment argues that noisy grading (or grade inflation) is a robust equilibrium phenomenon that often imposes a welfare cost on employers and other evaluators; none of these papers consider the interaction between grading policies and incentives to invest in education quality. Ostrovsky and Schwarz (2010) consider an assortative stable matching in a labor market. Vacancies are distinguished by desirability and graduates are distinguished by their expected ability. These authors argue that in equilibrium schools may not completely reveal the ability of their graduates to potential employers, assigning transcripts to students in a way that confounds employer beliefs about
graduate ability. Popov and Bernhardt (2013) consider a model of strategic grade assignment with a continuum of student abilities and two grades. They show that universities with better distributions of student abilities set lower grading standards; whereas a social planner would set a higher grading standard at a better university. Chan, Li, and Suen (2007) consider a game in which a school knows the distribution of its own students’ abilities, but an employer does not. In that paper, the proportion of good students at a given school can assume one of two values. A school can assign grades in a manner consistent with true student abilities, but in equilibrium, it sometimes inflates grades by assigning them as if the proportion of good students is high, even though it is actually low. If it could do so, a school would benefit from a commitment to honest grading.

Much of the grade assignment literature assumes (as we do) that evaluators can observe the grading policy utilized by a school prior to evaluating the graduates. Bar, Kadiyali, and Zussman (2012) consider the issue of transparency in grading, endogenizing the amount of information that a school makes public about its grading policies. In their model, students strategically choose courses with different difficulties and different degrees of grade inflation in order to affect employer perceptions about their abilities. They then contrast the impact of disclosing grading information to students prior to course selection and to employers along with transcripts. They find that disclosure of grading policies to students affects course selection decisions and that disclosing information to employers benefits students who elect to enter strictly graded courses and hurts those who select lenient courses. Overall, this can damage aggregate student welfare.

When investment is exogenous, we also find that grade inflation is a robust equilibrium phenomenon that hurts evaluator welfare, consistent with the main conclusions of the literature. However, because our focus is on the interaction between grading policy and investment, we reach different conclusions regarding the costs and benefits of allowing grade inflation (or strategic grade assignment). We show that allowing grade inflation motivates schools to invest in the ability of graduates and that these educational benefits can dominate the cost. Unlike the case in which graduate ability is exogenous, when schools invest to develop ability, graduates may be better-educated and the evaluator’s welfare may be higher, if noisy grading is permitted.

Several authors consider the teacher-student relationship in the principal-agent framework. In this interaction, the teacher’s goal is to incentivize student effort through the design of grading policies. By considering exam performance as a game of status4 Dubey and Geanakoplos (2010) demonstrate that students may be most motivated to exert effort when their exact exam performance is not revealed. Instead, it is more effective to publicly reveal performance information in broad categories (like letter grades). Related results are found in Zubrickas (forthcoming) who shows that if the market (or other subsequent evaluator) cannot observe an individual teacher’s grading practices, the teacher responds by grading leniently. Our analysis complements this strand of the literature. In both Dubey and Geanakoplos (2010).

4See Moldovanu, Sela, and Shi (2007).
and Zubrickas (forthcoming) the teacher initially commits to a grading policy in order to incentivize subsequent student effort. Meanwhile, in our work, a school’s initial investment decision determines its subsequent advantage in the grading stage game. The interaction between student effort and grading policy is closely related to the interaction between school investment and grading policy. While we do not explicitly address the issue of commitment to grading policy as a means of incentivizing student effort, we address the issue of student effort in Section VI.

Other authors consider issues to do with evaluation and certification but do not focus on the issue of grading standards. Taylor and Yildirim (2011) consider the interaction of an evaluator’s performance standard and an agent’s unobservable effort. They find that the evaluator often benefits by committing to ignore information about agent attributes during the assessment process. In their framework, agent effort is endogenous, but the evaluator’s signal structure is exogenous; this is in contrast to our framework which includes both endogenous agent effort (i.e., school investment) and signal structures (grading policies), as well as competition between agents. In the context of industrial organization, Lizzeri (1999) studies the incentives of a rating agency to disclose information about product quality. A seller with private information about quality has an opportunity to visit a rating agency with the capability to determine and certify quality. This rating agency commits to a disclosure rule (a stochastic mapping from qualities into reports) and to a price for this certification. The author shows that in a variety of important cases, the rating agency reveals a minimal amount of information to the market but appropriates a large share of the surplus.

Finally, our analysis is related to the concept of “signal-jamming,” often encountered in the industrial organization literature. In this literature, strategic agents distort their actions in order to interfere with another party’s inferences, “jamming” their ability to learn about some aspect of the strategic environment. In Fudenberg and Tirole (1986) an entrant faces uncertainty about the market’s profitability; an incumbent firm engages in predatory pricing to interfere with the entrant’s ability to learn. Holmström (1999) considers a model of dynamic managerial compensation in which a manager’s current performance reveals information about his ability, which influences his future wages. Early in his career, the manager exerts higher levels of unobserved effort in order to influence the market’s perception of his ability, increasing future wages. In our analysis schools deliberately introduce noise into their grading policies to manipulate the evaluator’s perceptions of graduate ability. Unlike standard models of signal-jamming, however, schools have direct control over their grading policies; they do not signal-jam by engaging in tertiary actions, like predatory pricing or effort exertion.

II. A Model of Strategic Grading and Investment

Consider a two-stage game between two schools and an evaluator. First, each school \( i \in \{\alpha, \beta\} \) invests in quality, which determines a graduate’s expected ability. Schools observe investments and then simultaneously choose grading policies that determine how transcripts are assigned to graduates of different abilities. Each school then produces a single graduate. The evaluator observes school investments,
graduate transcripts, and grading policies, and awards a single prize to a graduate from one of the two schools. The prize could be a desirable job, admission to a prestigious law school or university, or an elite scholarship; the evaluator could be a recruiter, admissions officer, or representative of a scholastic trust. The evaluator prefers to assign the prize to a “high-ability” graduate. Meanwhile, a school benefits whenever its graduate receives the prize, independent of his true ability.

In the first stage of the game, the schools simultaneously invest in education quality. Each school, \( i \in \{\alpha, \beta\} \), simultaneously chooses its quality level \( q_i \in [0, 1] \), which determines the ability distribution of its graduate. School \( i \)'s graduate is “high ability” \((\tau_i = h)\) with probability \( q_i \) and is “low ability” \((\tau_i = l)\) with probability \( 1 - q_i \). Since evaluators want to select high-ability students and schools want their students selected, schools benefit from investing in education quality. However, improving quality is also costly in terms of resources or effort: the cost of quality \( q_i \) for school \( i \) is \( C(q_i) = q_i^2 / \rho_i^2 \), where \( \rho_i \) determines the marginal cost of increasing school quality, with higher values corresponding to lower marginal costs. Parameter \( \rho_i \) may represent the availability of resources for the schools, for example, their infrastructure, endowment or donor base. Alternatively, it may represent characteristics of the student body, for example, its academic preparation. We allow for asymmetries in the parameter \( \rho_i \), reflecting differences along these dimensions: improving quality is less costly for school \( \alpha \), so that \( 0 < \rho_\beta \leq \rho_\alpha < 2 \).

Each school’s investment \( q_i \) is publicly observed at the end of the first stage.

In the second stage of the game, the schools simultaneously select grading policies. Before making this choice, schools observe quality investments \((q_\alpha, q_\beta)\), but they do not observe the realized ability of either graduate. A grading policy at school \( i \) is represented by a pair of conditional random variables \((H_i, L_i)\). The transcript of a high-ability student is an independent realization of \( H_i \) and the transcript of a low-ability student is an independent realization of \( L_i \). For technical reasons, we focus on random variables \((H_i, L_i)\) for which the cumulative distribution function has a finite number of discontinuities or mass points. Except at mass points, \( H_i \) and \( L_i \) admit differentiable densities with support over an interval. We refer to random variables with this structure as valid. Any pair of valid random variables that satisfies the monotone likelihood ratio property is an admissible grading policy. If an evaluator observes a transcript that is in the support of \( H_i \) but not in the support of \( L_i \), then the evaluator can correctly infer that the graduate receiving that transcript is high ability. Similarly, if an evaluator observes a transcript in the support of \( L_i \) but not in the support of \( H_i \), then she infers that the graduate must be low ability.

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5 Greenwald, Hedges, and Laine (1996) perform a meta-analysis, demonstrating a substantial link between school resources and student achievement, while Hanushek (2006) provides evidence that increased resources at schools do not necessarily translate into better educational outcomes. However, even those finding little evidence of the link between spending and performance do not necessarily claim that additional resources could not be beneficial. They claim that this link may not be observed strongly in the data because schools do not allocate their resources in the most effective way possible. As described in a previous footnote, Rivkin, Hanushek, and Kain (2005) demonstrate that teachers play an important role in educating students and may be costly to hire and retain.

6 Restricting attention to values of \( \rho < 2 \) is only to ensure that the marginal cost of education is sufficiently high that schools do not choose \( q = 1 \), guaranteeing a graduate of high ability.

7 In this context the monotone likelihood ratio implies that transcripts are ordered in such a way that a greater transcript realization is associated with a greater posterior belief that the graduate is high ability; that is, higher transcripts always brings “good news” about graduate ability.
If the evaluator observes a transcript in the support of both $H_i$ and $L_i$, then some uncertainty remains about whether the graduate is high or low ability. Given the prior beliefs, the school grading policy and the transcript realization, the evaluator’s posterior belief about ability is determined by Bayes’ rule.

This representation of a school’s grading policy is quite general and includes any possible system of grading that utilizes a finite number of letter grades; it also allows for more complex grading schemes such as assigning students a numerical value in the interval $[0, 100]$ or $[0, 4]$. Frequently, actual transcripts are not limited to a numeric or letter score, as they typically provide a list of classes taken by semester, grades by class, and overall grade point average. Some schools also include class/grade distribution on transcripts (see Bar, Kadiyali, and Zussman 2012). This is perfectly consistent with our model, with the random variables $H_i$ and $L_i$ together with the prior determining the likelihood that each possible transcript is owned by a high-ability graduate.

We can interpret the process of assigning grades as a process of disclosure, similar to the one in Lizzeri (1999). Following this interpretation, schools choose a grading policy (as described above) prior to learning the student’s ability. They then observe student ability and generate a transcript for their student in accordance with their set grading policies. Alternatively, we can interpret a transcript as the verifiable outcome of a “test” that is designed by the school and administered to students during the grading process. In this way, grading can be viewed as a process of Bayesian Persuasion, as described by Kamenica and Gentzkow (2011).

Once investments and grading policies are set, graduate abilities and transcripts are realized. Each school’s graduate is evaluated and the prize is awarded. Before awarding the prize, the evaluator observes the quality of each school, the grading policy at each school, and each graduate’s realized transcript. If she awards the prize to a high-ability graduate, she receives payoff one. Otherwise, her payoff is zero. The evaluator’s expected payoff of offering the prize to each graduate is equal to the posterior probability that the graduate is high ability. It is therefore sequentially rational for her to offer the prize to the graduate whom she believes is more likely to be high ability. We assume throughout that if she holds the same beliefs about each graduate then she randomizes fairly between them. A school’s payoff is one if its graduate is awarded the prize, and it is zero otherwise. Once the prize is awarded, the true type of the recipient is revealed and payoffs are realized.

### A. Preliminaries

Before moving on to the equilibrium characterizations, we present two preliminary results that help to streamline subsequent analysis.

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8 In this interpretation, students’ true abilities are verifiable. In this way, an outsider, perhaps an accreditation agency, has the ability to monitor the school’s grade assignments to ensure that it adheres to its stated disclosure rule. While the evaluator would like to demand that the school release graduate’s true abilities in the second stage, we assume that a school’s choice of a grading policy is binding.

9 We assume that awarding the prize to a low-ability graduate dominates assigning the prize to no one. This assumption is not necessary for our results, but streamlines the exposition drastically. We also believe that this assumption is realistic in a variety of settings.
Bayesian Persuasion Representation.—Following the approach of Bayesian Persuasion developed by Kamenica and Gentzkow (2011), we describe a representation of grading policies that considerably simplifies the analysis of the grading stage of the game. At the end of the grading stage, the evaluator observes the student’s transcript and updates her beliefs about each graduate’s ability taking into account both the school’s grading policy and its investment level. At the beginning of the grading stage, schools commit to generate transcripts for high-ability graduates as realizations of random variable $H_i$ and low-ability graduates as realizations of random variable $L_i$. If these random variables are not identical, then by considering the relative likelihood of a certain transcript being assigned to graduates of each ability (and the prior belief about the graduate), the evaluator draws an inference about ability from the transcript. In equilibrium, the evaluator updates beliefs while accounting for the transcripts she observes, the method by which these transcripts are assigned and the school’s investment level, which was selected by the school in stage one and represents the prior belief that the graduate is high ability. The school’s grading policy thus affects both the probability of each specific transcript realization and the inference that an evaluator draws from observing any specific transcript. Therefore, from a school’s perspective, its choice of grading policy induces a random variable for the evaluator’s posterior belief. We can therefore represent any feasible grading policy $(H_i, L_i)$ by a single random variable $\Gamma_i$ from which the evaluator’s posterior belief about a graduate’s ability is drawn.

Formally, suppose that given the prior belief about student ability, transcript realization $x$ from grading policy $(H_i, L_i)$ results in the evaluator having posterior belief $\gamma = \Pr(t = h|x)$. Along with the prior, grading policy $(H_i, L_i)$ also determines the probability distribution of $x$, which is itself a random variable $X$. Thus, the prior belief and grading policy determine the ex ante distribution of the evaluator’s posterior belief, $\Gamma_i = \Pr(t = h|X)$. Random variable $\Gamma_i$ is valid, has support confined to the unit interval and, according to the Law of Iterated Expectations, has expectation equal to the prior, $q_i$. In the next lemma (adapted from Kamenica and Gentzkow 2011), we show that these are the only substantive restrictions on the ex ante posterior beliefs that can be generated by a grading policy.

**Lemma 1 (Bayesian Persuasion Representation):** Consider any valid random variable $\Gamma_i$ with support confined to the unit interval and expectation $q_i$. If the prior belief that a student is high ability is $q_i$, then there exists a grading policy $(H_i, L_i)$ for which the ex ante posterior belief is $\Gamma_i$.

This lemma simplifies the analysis considerably. All payoff-relevant aspects of a grading policy are summarized by a single random variable representing the ex ante distribution of the evaluator’s posterior belief, and any random variable with support inside $[0, 1]$ and mean equal to the prior is the ex ante posterior belief for some grading policy. The analysis can therefore focus on an alternative version of our

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10 Random variable $X$ is a mixture of $H_i$ and $L_i$, in which $H_i$ is weighted by the prior probability that the graduate is high ability, $q_i$.

11 In fact, large class of grading policies.
original game in which each school chooses \( \Gamma_i \) rather than \((H_i, L_i)\), as long as \( \Gamma_i \) is valid, has support in the unit interval, and expectation \( q_i \). We refer to the choice of \( \Gamma_i \) as a choice of a grading policy; although \( \Gamma_i \) technically represents an entire pay-off-equivalent class of grading policies.

A school would like \( \Gamma_i \) to only generate high realizations of the posterior belief. This is not feasible, however, because Bayesian rationality requires that the expected value of the posterior belief generated by any grading policy must be equal to the prior probability \( q_i \). Any probability mass generating high beliefs (greater than \( q_i \)) must be offset by a probability mass generating low beliefs (below the prior). The freedom to choose a grading policy allows a school to strategically control information about the ability of its graduates, but it cannot use its grading policy to make its graduate appear to be better, on average, than he truly is. Therefore, having a high investment in quality \( q_i \) benefits a school by allowing it to generate more favorable belief distributions.

**Evaluator Strategy and Beliefs.**—Evaluation takes place at the end of stage two. At this point in the game the evaluator updates her beliefs about graduate abilities given all available information—investments, grading policies, and transcripts. This process generates realizations \((\gamma_\alpha, \gamma_\beta)\) of the posterior belief random variables \((\Gamma_\alpha, \Gamma_\beta)\). The evaluator then awards the prize. Because she receives payoff one if she assigns the prize to a high-ability graduate and zero otherwise, it is sequentially rational for her to assign the prize to the graduate that she believes is more likely to be high ability.

**Lemma 2** (Sequentially Rational Evaluator Strategy and Evaluator Beliefs): *For any combination of beliefs, it is sequentially rational for the evaluator to assign the prize to the graduate whom she believes is more likely to be high ability and to randomize fairly when her beliefs are identical. At the evaluator’s information set, her beliefs about graduate abilities are the realizations \((\gamma_\alpha, \gamma_\beta)\) of the posterior belief random variables \((\Gamma_\alpha, \Gamma_\beta)\).*

This result characterizes the evaluator’s equilibrium strategy in the final stage of the game. We refer to this result when describing the Perfect Bayesian Equilibrium of the entire game in Proposition 4.4 below.

**III. Equilibrium with Strategic Grading**

We characterize the unique Perfect Bayesian Equilibrium of the game with strategic grading using backwards induction. First, we characterize the unique equilibrium of the grading stage, following any combination of school investments chosen in the investment stage and discuss some of its properties. We then use the payoffs from this second stage, as functions of school investments, to derive the unique equilibrium investment levels in stage one.

**Stage Two: Grading.**—In the grading stage of the game, educational investments \((q_i, q_j)\) are taken as given, because these were chosen in stage one. The schools
simultaneously design grading policies, which generate posterior belief random variables \((\Gamma_i, \Gamma_j)\). As described in Lemma 1, we allow the schools to choose the posterior belief random variables directly, provided their supports are confined to the unit interval, and the expected value of the posterior belief random variable is equal to the prior: \(E[\Gamma_i] = q_i\). In the process of evaluation, these random variables are realized, and (as described in Lemma 2) the evaluator awards the prize to the graduate whose posterior belief random variable generates a larger realization, randomizing fairly in case of ties.\(^{12}\)

Anticipating sequentially rational prize assignment by the evaluator, school \(i\)'s expected payoff when schools choose grading policies \((\Gamma_i, \Gamma_j)\) is equal to \(\Pr(\Gamma_i > \Gamma_j) + (1/2)\Pr(\Gamma_i = \Gamma_j)\). With the Bayesian Persuasion representation of grading policies, the evaluation stage of the strategic grading stage game resembles a full information all-pay auction with the inclusion of mixed strategies: in both of these games players design random variables, and the player whose realization is higher wins a prize of known value. There are two fundamental differences between the evaluation stage of our game and a standard full information all-pay auction. First, the “bid distribution” in our environment is constrained to a support inside the unit interval (because a realization represents a probability). Second, the bid distribution must maintain an expected value equal to \(q_i\) (the average ability of a graduate). This second distinguishing feature also appears in Wagman and Conitzer (2011), which analyzes an all-pay auction with a constraint on the mean bid.\(^{13}\)

While the strategic grading stage is closely related to this version of the all-pay auction, the focus of our analysis is on the interaction between incentives to invest in education quality (in stage one) and a school’s freedom to design grading policies strategically (in stage two). This comparison has no counterpart in Wagman and Conitzer (2011): there is no analog of “fully informative grading” in their analysis, or any other variation of the strategic environment that we consider (uninformative grading, student effort).\(^{14}\) Therefore, the main contributions of our analyses are significantly different.

Suppose that in the investment stage, one of the schools, referred to as \(A\), invests more in school quality than the other school, referred to as \(B\). That is, the stage one investment levels are \((q_a, q_b)\) where \(q_b \leq q_a\). Because school \(A\) has invested more, we refer to \(A\) as the high-quality school.\(^{15}\) The following lemma characterizes the grading strategies that comprise an equilibrium of the second stage subgame.

\(^{12}\) In equilibrium ties can only occur when both realizations reveal the graduates to be high ability for certain, \(\gamma_\alpha = \gamma_\beta = 1\).

\(^{13}\) A similar constraint appears in Dulleck, Frijters, and Podczeck (2006).

\(^{14}\) It is important to point out that Wagman and Conitzer (2011) do consider an initial stage in which the mean bid is determined by an investment with a linear cost. However, because costs in their model are linear, their two stage game is outcome equivalent to a standard all pay auction.

\(^{15}\) As one would expect, when we derive the equilibrium of the investment stage, the advantaged school \(\alpha\) will choose to invest more in education and will play the role of school \(A\) on the equilibrium path.
Lemma 3 (Equilibrium of Grading Subgame): For each possible combination of stage one investments \((q_a, q_b)\), the unique equilibrium of the corresponding second-stage subgame is given by the following grading policies \((\Gamma_a, \Gamma_b)\):

- When \(q_a \leq 1/2\):

  \[
  \Gamma_a = U[0, 2q_a] \quad \Gamma_b = \begin{cases} 
  0 & \text{with prob } 1 - \frac{q_b}{q_a} \\
  U[0, 2q_a] & \text{with prob } \frac{q_b}{q_a}
  \end{cases}
  \]

- When \(q_a > 1/2\):

  \[
  \Gamma_a = \begin{cases} 
  U[0, 2(1 - q_a)] & \text{with prob } \frac{1}{q_a} - 1 \\
  1 & \text{with prob } 2 - \frac{1}{q_a}
  \end{cases}
  \Gamma_b = \begin{cases} 
  0 & \text{with prob } 1 - \frac{q_b}{q_a} \\
  U[0, 2(1 - q_a)] & \text{with prob } \frac{q_b}{q_a} \left(\frac{1}{q_a} - 1\right) \\
  1 & \text{with prob } \frac{q_b}{q_a} \left(2 - \frac{1}{q_a}\right)
  \end{cases}
  \]

- In both cases:

  \[
  u_a(q_a, q_b) = 1 - \frac{q_b}{2q_a} \quad \text{and} \quad u_b(q_a, q_b) = \frac{q_b}{2q_a}.
  \]

When \(q_a \leq (1/2)\), both schools are more likely to produce a low-ability graduate than a high-ability graduate. In this situation, the school with the quality advantage chooses a grading policy that leaves the evaluator less than fully informed about the quality of its graduate, employing a grading policy that generates a uniform distribution over posterior beliefs centered on the prior. Thus, every transcript that the high-quality school assigns can be assigned to both a high-ability or low-ability graduate. By always maintaining a mix of both types for any given transcript, the school utilizes its (unlikely) high-ability graduate to improve the perception of its (likely) low-ability graduate. In equilibrium, school \(B\) adopts a grading policy that mimics the grading policy of school \(A\) with one exception: in order to maintain \(E[\Gamma_b] = q_b < q_a\), school \(B\) sometimes issues a transcript that reveals (for certain) that its graduate is low ability. The low-quality school would like to neutralize school \(A\)'s initial advantage by mimicking its grading policy exactly, but cannot because Bayesian rationality on the part of the evaluator means that the ex ante expected quality at school \(B\) must be lower. To maintain the credibility of its grading, the low-quality school sometimes issues a low-ability student a transcript that fully
reveals his type, but otherwise utilizes the same grading policy as the high-quality school.\footnote{In the Appendix, we argue that this mimicry exists not only on the level of the posterior distribution, but also exists on the underlying grading policy. In order to achieve the equilibrium posterior belief distribution school $B$ can use a grading policy $(H_b, L_b)$ that is identical to school $A$’s, $(H_a, L_a)$ except that $L_b$ contains a special transcript realization (or set of realizations) that is issued only to low-ability students.}

When $q_a > (1/2)$, the higher quality school is more likely to produce a high-ability graduate than a low-ability graduate. In this case, the higher quality school does not need to assign every transcript to a mix of both high- and low-ability graduates. Like the previous case, school $A$ never reveals that its graduate is low ability, but unlike that case, it sometimes reveals that its graduate is high ability. To do this, the school reserves some “outstanding” transcripts (i.e., 4.0 GPA, honors program, etc.) for only high-ability graduates and all other transcripts are assigned to a mix of both high- and low-ability graduates. The lower quality school again responds by sometimes revealing when its graduate is low ability, but otherwise mimicking the posterior belief distribution (and underlying grading policy) of the other school.

The main properties of the (subgame) equilibrium grading strategies are summarized in the following proposition. These properties hold for any stage one investments $(q_a, q_b)$.

**PROPOSITION 1 (Properties of Grading Policies):** For each possible combination of stage one investments $(q_a, q_b)$, where school $A$ invests more than school $B$ (i.e., $q_b \leq q_a$), the grading policies that arise in equilibrium of the corresponding grading subgame have the following properties:

(i) Neither school uses a fully informative grading policy.

(ii) School $A$ does not reserve any transcripts realizations for low-ability students. All transcripts that may be assigned to low-ability students may also be assigned to high-ability students.

(iii) School $B$ reserves certain transcripts for only low-ability students; otherwise it uses the same grading policy as school $A$.

(iv) The grading policy at school $B$ is more Blackwell informative than the grading policy at school $A$.

Consistent with previous literature on grade assignment, we find that fully informative grades are not part of an equilibrium for any investment levels. Furthermore, we find that schools whose students are less likely to be high ability use grading policies that are more informative.

Although school $A$ has a better education quality, if school $B$ were able to perfectly mimic school $A$’s posterior belief distribution, it could completely neutralize school $A$’s initial advantage, guaranteeing a payoff of $1/2$ for both schools. However, because the prior belief about ability is lower at school $B$, school $A$’s posterior belief
distribution is not feasible, because no rational evaluator would believe that the average ability of graduate $B$ is the same as graduate $A$. In order to stay competitive, school $B$ sometimes reveals that its graduate is low ability—effectively conceding the prize—which restores enough of its credibility for $B$ to plausibly mimic $A$’s posterior belief distribution the rest of the time. The greater the investment gap between schools, the more often $B$ is forced to concede. This effect arises because strategic grading is noisy and school investment (the evaluator’s prior) always plays a direct role in the evaluator’s inference.

Our analysis identifies a grading policy with a distribution of posterior beliefs that it induces. In this way, our analysis identifies the informational content of equilibrium grades, but cannot distinguish among the many nominal grading policies that could convey this information (including both inflationary and deflationary grading policies). However, sophisticated evaluators would not respond to changes that occur only in the nominal grade assignment rule (for example, a constant upward shift in GPA or nominal transcript for all abilities simultaneously); it is the informational content of grades that is important for evaluator decisions, which influence schools’ incentives for investment in education.

Stage One: Investment in School Quality.—In the first stage of the game, schools simultaneously invest in education quality, which affects the expected ability of their graduate. Recall that school $\alpha$ has a lower marginal cost of improving education quality than school $\beta$: $0 < \rho_\beta \leq \rho_\alpha$.

Anticipating the second-period equilibrium grading policies induced by a combination of stage one investments, in stage one each school $i \in \{\alpha, \beta\}$ expects payoff $u_i(q_\alpha, q_\beta)$ from combination of investments $(q_\alpha, q_\beta)$, where

$$u_i(q_\alpha, q_\beta) = \begin{cases} \frac{q_i}{2q_j} - \frac{q_i^2}{\rho_i^2} & \text{if } q_i \leq q_j \\ 1 - \frac{q_j}{2q_i} - \frac{q_j^2}{\rho_j^2} & \text{if } q_i > q_j. \end{cases}$$

This function is differentiable, continuous and concave in $q_i$. The equilibrium investment levels are presented in the following lemma.

**LEMMA 4 (Strategic Investment):** Anticipating the strategies and payoffs of the grading subgame (described in Lemma 3), in the unique equilibrium of the investment stage, schools choose investments $q_\alpha = \frac{\sqrt{\rho_\alpha \rho_\beta}}{2}$ and $q_\beta = \frac{\sqrt{\rho_\alpha \rho_\beta}}{2} \frac{\rho_\beta}{\rho_\alpha}$.

School $\alpha$’s equilibrium investment is an increasing function of the product $\rho_\alpha \rho_\beta$. School $\alpha$ invests more when $\rho_\alpha$ increases (reducing its marginal investment cost, a direct effect) or in response to competitive pressure, a strategic effect generated by an increase in $\rho_\beta$ that reduces $\alpha$’s initial advantage. If $\rho_\alpha$ is reduced, the strategic and
direct effects work in opposite directions: investment is more costly for \( \alpha \), but competition is stronger, because its advantage is smaller. In this case, the direct effect dominates, and \( \alpha \)'s investment shrinks as its marginal cost of investment increases. School \( \beta \)'s equilibrium investment is the same as school \( \alpha \)'s, except that it is scaled down by \( \rho_\beta / \rho_\alpha \), a consequence of the higher marginal investment cost that school \( \beta \) faces. School \( \beta \) invests more when \( \rho_\beta \) increases, as its marginal cost is lower and competitive pressure is higher. An increase in \( \rho_\alpha \) reduces \( \beta \)'s investment because it reduces competitive pressure.

The following proposition summarizes our characterization of the equilibrium of the grading and investment game, when grading is strategic.

**PROPOSITION 2** (Equilibrium with Strategic Grading): The unique Perfect Bayesian Equilibrium of the strategic grading and investment game is given by investment levels \((q_\alpha, q_\beta)\) described in Lemma 4, the grading policies described in Lemma 3 (for all possible investment combinations), and the evaluator strategy and beliefs described in Lemma 2.

### IV. Benchmark: Fully Informative Grading

In this section, we describe the equilibrium outcomes when evaluators always perfectly observe graduate ability; we refer to this as the **fully informative grading benchmark**.\(^{17}\) Fully informative grading may result from commitment by schools to strong grading standards, or from rules imposed by university accreditation agencies. It could also be a consequence of requiring university exit exams, graduate school admissions exams or industry licensing exams given to all graduates applying for a position. The more comprehensive the exam, the greater its potential to distinguish student abilities. In principle, such exams may be sufficiently informative to fully reveal student types, making transcripts irrelevant.\(^{18}\)

When abilities are fully revealed, school \( i \)'s graduate receives the prize with probability \( 1/2 \) when the realized abilities of the graduates are the same, and with probability 1 when the graduate is high ability and his competitor is low ability. School \( i \)'s expected payoff given school quality investments \((q_i, q_j)\) is therefore

\[
\begin{align*}
    u_i(q_i, q_j) &= q_i(1 - q_j) + \frac{1}{2}(q_i q_j + (1 - q_i)(1 - q_j)) - \frac{q_i^2}{\rho_i^2} \\
    &= \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho_i^2}.
\end{align*}
\]

\(^{17}\)Formally, for both schools, the set of possible transcript realizations for high-ability students and the set of possible transcript realizations for low-ability students do not overlap. This generates posterior belief random variables that are Bernoulli: \( \Pr(\Gamma_i = 1) = q_i \) and \( \Pr(\Gamma_i = 0) = 1 - q_i \). Realization 1 reveals that the graduate is high ability for certain, while realization 0 reveals low ability. Note that in this benchmark, grading policies are not chosen by the schools.

\(^{18}\)A number of nonprofit and for-profit entities have begun to offer exit exams at the university level as a means of conveying (or eliciting) more information about graduate ability than can be gleaned from transcripts. Describing one such exam, the CLA+, David Pate, Dean of the College of Arts and Sciences at St. John Fisher College, claims that the test “provides an objective, benchmarked report card for critical thinking skills” (Belkin 2013).
As is evident from the above expression, the marginal benefit of improving quality for either school is independent of the other school’s investment. If the other school’s graduate turns out to have low ability, a marginal increase in investment reduces the probability of ties, increasing the probability of winning the prize, resulting in a net marginal benefit of $1 - 1/2 = 1/2$. Similarly, if the other school’s graduate turns out to be high ability, then a marginal increase reduces the probability of losses and increases the probability of a tie, resulting in a net benefit of $1/2 - 0 = 1/2$. Thus, under fully informative grading, the marginal benefit of improving school quality is equal to $1/2$ for each school, independent of the other’s investment. It is therefore not surprising that when schools choose investment in stage one, each school has a dominant strategy.

**PROPOSITION 3** (Equilibrium with Fully Informative Grading): The unique Perfect Bayesian equilibrium of the fully informative grading game is given by investment levels $q_\alpha = \rho_\alpha^2/4$ and $q_\beta = \rho_\beta^2/4$ and the evaluator strategy described in Lemma 2. At each information set, the evaluator knows both graduates’ abilities with certainty.

In the next sections we compare educational investment and evaluator payoffs with strategic grading to this benchmark.

**V. Strategic versus Fully Informative Grading**

In this section, we compare equilibrium outcomes in the game when schools strategically choose grading policies to equilibrium outcomes in the fully informative benchmark. If school quality is exogenous (the standard assumption in the literature), then requiring fully informative grades makes the evaluator better off compared to strategic grading. With exogenous quality, the evaluator’s payoff depends only on her ability to select high-ability graduates. Because strategic grading is noisy, the evaluator would benefit by requiring that grades completely reveal graduate ability.

We compare outcomes when school quality is endogenous. Here, we find that allowing strategic grading brings surprising benefits: when schools grade strategically they often invest more to improve education quality. This benefit also may reverse the evaluator’s welfare ranking from the case of exogenous qualities; with endogenous investment, the evaluator prefers strategic grading to fully informative grading if the school’s investment costs are not too different. Even if she had the capability to eliminate all noise from student transcripts, it may not be in her interest to do so. Our first main result concerns school investments:

**PROPOSITION 4** (Strategic Versus Fully Informative Grading: Investment): Requiring fully informative rather than strategic grading always decreases school $\beta$’s education quality and decreases school $\alpha$’s education quality if and only if $\rho_\beta \in \left((1/4)\rho_\alpha^2, \rho_\alpha\right)$.

Schools often invest more in quality when they grade strategically. To develop intuition for this result, recall that with fully informative grading the marginal
benefit of increased investment for either school is fixed and equal to $1/2$.\footnote{As discussed in Section IV, marginally increasing investment results causes fewer ties in which both schools produce low-ability graduates and allows the investing school to win more often, and also causes more ties in which both schools produce high-ability graduates, creating ties in situations in which the school would have lost for certain.} With strategic grading, however, the marginal benefit of investment is not fixed at $1/2$ for either school. As discussed previously, by closing the investment gap, the lower quality school, $B$, reduces the probability of revealing that its graduate is low ability, contesting the prize more often. In equilibrium, the prize is contested with probability equal to $q_b/q_a$. If it does not reveal a low-ability graduate, school $B$ mimics the grading policy of school $A$, giving it an expected payoff of $1/2$ in this circumstance. Thus, for school $B$, the marginal benefit of increased investment in quality is $1/(2q_a)$. This marginal benefit of investment is fixed (because the probability of contesting the allocation of the prize is linear in $q_b$) and is greater than the marginal benefit of investment with fully informative grades. The disadvantaged school, $\beta$, therefore always invests more with unrestricted strategic grading than with fully informative grading, in order to avoid conceding the prize to $\alpha$ as often. Conversely, school $A$ benefits from widening the investment gap by forcing school $B$ to concede the prize more often in equilibrium. However, the probability that school $B$ concedes the prize, $1 - q_b/q_a$, depends on $q_a$ in a concave way. The marginal benefit of increasing quality for school $A$, $q_b/(2q_a^2)$, is diminishing. Thus, depending on the marginal costs of increasing quality, school $\alpha$ may choose either higher or lower investment when grading is unrestricted. If the schools are similar ex ante, then competition is most-fierce and both schools invest more in education quality.

From the evaluator’s perspective, strategic grading introduces a tradeoff between ex post selection of a high-ability graduate and ex ante investment to generate a high-ability graduate. Although with strategic grading the evaluator is less able to determine each graduate’s ability, because strategic grading leads to greater education investment, the evaluator may be better off. It is a simple matter to check that whenever the marginal costs of investment are identical the evaluator prefers strategic grading to the fully informative benchmark; even with asymmetric costs a region exists around the diagonal in which this result holds.

**PROPOSITION 5 (Strategic Versus Fully Informative Grading: Evaluator Welfare):**

*Requiring fully informative rather than strategic grading hurts the evaluator whenever the initial asymmetry between schools is not too large. For each value of $\rho_\alpha$, there exists a value $\bar{\tau} < \rho_\alpha$, such that, if $\rho_\beta \in (\bar{\tau}, \rho_\alpha)$, then the evaluator’s equilibrium payoff is lower with fully informative grading policies.*

Allowing schools the freedom to grade strategically can benefit the evaluator. Eliminating grade inflation can decrease both the average ability of graduating students and the ex ante probability that the evaluator awards the prize to a high-ability graduate.
VI. Student Effort

In the analysis we present above, the student is a passive participant in the learning process: only a school’s level of investment affects his likely educational outcome. It is natural to think that students actively participate in their own education by exerting effort. This section considers an alternative version of the game that extends the original framework, allowing graduate quality to depend on both school investment and student effort. We present a summary of our analysis here. The full analysis is provided in the online Appendix.

We augment the model of Section II by incorporating two strategic students, one for each school. The ex ante probability that graduate $i$ has high ability, $Q_i$, is determined by school $i$’s investment, $q_i \in [0, 1]$, and student $i$’s effort, $e_i \in \{0, 1\}$. We focus on an educational process in which school investments translate into graduate ability, provided that students exert the necessary level of effort. If a student does so, then his school’s educational investment determines his likely ability; otherwise, the student’s ability is low. Thus, student effort and school investments jointly determine graduate ability, with

$$Q_i = q_i e_i.$$ 

In order to exert high effort, a student bears cost $\kappa > 0$. For simplicity, we focus on the case of symmetric schools, so that $\rho_i = \rho_j = \rho$, although this is not essential for the results. Like schools, students value the prize at one, regardless of their ability.

In the first stage of the game, school investments and student effort are chosen simultaneously. School investments are publicly observed, but student efforts are not observable by anyone other than the student. Hence, schools and the evaluator act on their conjectures of the student’s strategy (in equilibrium these conjectures are correct); let $\hat{e}_i$ denote the schools and evaluator’s conjecture about student $i$’s effort level. Along with observable school investments, conjectures about student efforts determine all parties’ prior beliefs about graduate ability, $Q_0 = q_i \hat{e}_i$. In the second stage, schools simultaneously choose grading policies $(H_i, L_i)$ as described in Section II; however, as we explain below, the connection between grading policies and the posterior belief distribution is more subtle with unobserved effort. Next, each graduate’s transcript is realized. Given her prior belief about graduate ability—now a function of school investment and conjectured student effort—the grading policies and observed transcripts, the evaluator updates her beliefs about each graduate’s ability and assigns the prize to the graduate she believes is more likely to be high ability.

An equilibrium with complete coordination failure always exists: if students are expected to always shirk, $\hat{e}_i = \hat{e}_j = 0$, then anticipated graduate quality is $Q_0 = 0$ regardless of school investment (or grading policy). Thus, the prize is assigned to each graduate with probability $1/2$. If the prize is always assigned randomly, then zero investment by schools and zero effort by students are best responses.

We next consider the possibility of an equilibrium in which students exert effort for certain. Suppose that schools and the evaluator expect students to exert effort for
certain, $\hat{e}_i = 1$. In the online Appendix (see Lemma B.3), we show that in this case school investments, grading policies, evaluator beliefs, and prize assignments are all identical to those described in Section III. Intuitively, the schools and the evaluator all act on their conjectures that students are exerting effort, implying that the probability that a graduate is high ability is determined solely by his school’s investment, exactly as in Section III. Thus, schools invest $q_i = q_j = \rho/2$ and schools design grading policies $(H_i, L_i)$ in the grading subgame that they anticipate will generate the posterior belief random variables in Lemma 3, under the conjecture that $\hat{e}_i = \hat{e}_j = 1$.

If a student privately shirks in the investment stage, the school does not observe this deviation in student effort, and the school will therefore not change its investment or grading policy in response to the shirking. However, because the student’s transcript is always a realization of random variable $L_i$ (instead of a mixture between $L_i$ and $H_i$ in which $H_i$ is realized with probability $q_i$), the true distribution of the posterior belief generated by the grading policy is different than the one the school expects. Indeed, we find that if the school designs the grading policy to generate a posterior belief random variable with density $g(x)$ under the conjecture that the student exerts effort, then a shirking student’s posterior belief random variable will have density $g(x)(1 - x)/(1 - q_i)$. The true density places more mass on low realizations of the posterior belief and less mass on high realizations. By shirking, the student saves on the effort cost, but he secretly induces a less favorable posterior belief distribution than would arise if he exerted effort.

The unfavorable change in the posterior belief distribution resulting from shirking is more damaging to the student when schools are expected to invest significantly in education. If school investments are high, then by shirking the student puts himself at a greater disadvantage, as his competitor is more likely to have high ability and is thus more likely to generate a high value of the posterior belief. Thus, the cost of shirking increases with the level of expected school investment. In this sense, anticipated school investment and student effort exhibit strategic complementarity: the more investment is expected, the higher is the threshold effort cost below which the student prefers to exert effort instead of shirking for certain. Thus, student effort is incentive compatible whenever the effort cost $\kappa$ is below some threshold $\phi(\rho)$, that is increasing in school resources. Above this threshold, only the equilibrium with student shirking and zero investment exists.

Next, we consider equilibria of the fully informative grading game with student effort. As above, if students are expected to exert effort, then the equilibrium investment levels and evaluator acceptance decisions are identical to the case without student effort. For student effort to be incentive compatible, effort cost must again lie below some threshold $\phi^*(\rho)$, which is increasing in school resources. If the cost exceeds the threshold, then only the equilibrium with students shirking and zero school investments exists. In the online Appendix (see Proposition B.5), we show that $\phi(\rho) > \phi^*(\rho)$, and therefore the equilibrium with strategic grading exhibits (weakly) higher effort than the fully revealing grading equilibrium.

This is a counterintuitive result. Unobserved effort introduces moral hazard for students; when the signal generated by transcripts is less informative about their ability, the incentive to shirk is stronger. It is therefore plausible that student effort
would be lower with strategic grading. Indeed, holding school investment fixed, students are more inclined to shirk with strategic grading. However, with strategic grading, a countervailing effect arises. Strategic grading generates higher levels of investment in education quality by schools, which motivates student effort through strategic complementarity. This effect dominates the moral hazard: students exert weakly higher effort with strategic grading because strategic grading strengthens incentives for school investment.

The previous results and their implications are summarized in the following proposition, which contrasts equilibrium outcomes arising under strategic grading and fully informative grading, establishing that the main results from Section V comparing school investment and evaluator payoffs are robust to the inclusion of unobserved, endogenous student effort in the model (see Propositions B.5 and B.6 in the online Appendix).

**Proposition 6** (Strategic Versus Fully Informative Grading with Student Effort): In the model presented in Section 7, requiring fully informative rather than strategic grading (i) never increases and sometimes decreases student effort, (ii) never increases and sometimes decreases school investments, and (iii) never benefits and sometimes hurts the evaluator.

**VII. Additional Discussion**

**Competitive Structure.**—In our model, investment in education is driven by school competition: with a single school, its graduate would be placed for certain regardless of its investment or grading policy. Hence, the school would invest nothing and the graduate would be low ability. With competition in the absence of fully revealing grading standards, part of the benefit of educational investment for a school is inherently strategic: by investing more in education, a school can adopt a more favorable position prior to the beginning of the grading stage of the game. This effect is largest, however, when the number of schools feeding graduates to a particular evaluator is relatively small. If many schools compete for placement such that the strategic effect is negligible, then schools have no incentive to invest more with strategic grading than with fully revealing grading, and strategic grading hurts evaluator welfare. Thus, our model and analysis apply best to settings in which evaluators recruit from a relatively small number of schools, where the strategic interaction of grading and educational investment is most relevant. Evidence analyzed by Rivera (2011) suggests that this often occurs when the prize is employment at elite firms that tend to solicit and screen candidates from only a small subset of schools. Anecdotal evidence also points to similar effects in graduate school admissions. In these cases, the schools whose graduates are considered by the evaluator tend to be similar in terms of their levels of resources, in which case strategic grading is likely to dominate fully informative grading.

\[^{20}\\text{For example, in an article on CNNMoney, the author concludes "What the data strongly suggests is that getting into [the M.B.A. program under consideration] is a name game where the doors are either not as open or just plain closed to alumni of less prestigious companies and schools." See John A. Byrne, August 8, 2011, http://tinyurl.com/l7qxawk.}^{20}\]
**Quadratic Costs.**—In order to derive relatively simple closed form solutions for all variables of interest, we have focused on the case in which the cost of improving expected student ability is quadratic. In the body of the paper, we have described the intuition, as much as possible, only in terms of marginal benefit; the reasoning and intuition in the paper therefore also apply for other strictly convex cost functions. While strict convexity is a standard assumption, we also point out that if costs were linear, the two stage game would be essentially outcome-equivalent to a setting with uninformative grades.

**Grading Control.**—In our model, schools control grading policies. This may be a reasonable assumption at a number of institutions, where administrators impose certain restrictions on the nominal grade assignment rules, which may translate into restrictions on information provision. At other institutions, faculty may have autonomy in their choice of grading policies. To the extent that school administration anticipates that faculty will choose strategic or noisy grading policies, similar incentives to increase investments in education quality exist. This suggests that the deterioration of strict grading standards may incentivize increased investment in school quality and lead to higher evaluator welfare, even when the grading policies are not directly controlled by school administration.

**Commitment to Fully Revealing Grades.**—Comparing a framework with strategic grading to a framework in which grading is necessarily fully informative, we show that an evaluator may benefit from allowing strategic grading, even when she has the ability to require fully informative grading by both schools. Suppose that instead of an evaluator requiring fully informative grading, schools can commit before investment decisions are made to provide a fully informative assessment (for example, by committing to administer rigorous exit exams). To explore this possibility, the online Appendix considers an augmented game with an initial period, in which schools can simultaneously and publicly commit to fully informative grading. After any commitments are observed, the game proceeds exactly as in Section III, except that a school that committed must issue fully revealing grades in the second stage (while a noncommitting school is not bound by this restriction). To simplify the analysis, we assume symmetric schools (with common $\rho$) and no student effort. We characterize the equilibrium of the game with the commitment stage.

When school resources, $\rho$, are below a threshold, the commitment stage becomes a prisoner’s dilemma between schools: (not commit) is a dominant strategy for both schools, though both schools are better off if they would play (commit, commit). In this range of parameters, schools’ ability to commit to fully informative grading has no effect on the equilibrium outcome. For intermediate levels of school resources, two asymmetric pure strategy equilibria exist. In each equilibrium, one school commits to fully informative grading and the other does not. Compared to the case when neither school commits, when one school commits it softens investment stage competition by enough that it is better off compared to no commitment. For high levels of school resources, commitment by both schools is a dominant strategy equilibrium. In this range, the existence of the commitment device clearly hurts the evaluator.
Evaluator Rationality.—In our model, the evaluator considers the information content of grades when assessing graduates. She derives her updated beliefs about graduate ability from Bayes’ rule, given the nominal grading policy actually utilized by his school. In some cases, however, evaluators may exhibit a psychological bias toward the fully informative grading policy, mistakenly overestimating the good news communicated by a high transcript realizations, without taking the actual grade assignment rule fully into account. While evaluators who systematically make this type of error would likely be forced to adapt in the long run, it is important to point out that if this type of error is pervasive then the popular concerns about the erosion of grading standards are more relevant. Indeed, if all evaluators incorrectly assume that any graduate with a high GPA must be high ability, then schools will assign these transcripts liberally and invest nothing in education. Our analysis suggests that, if such a psychological bias is indeed common, then a policy that corrects this bias could (in some cases) do more to improve education quality and evaluator welfare than a policy that strengthens grading standards.

VIII. Conclusion

We use a sequential model of school competition, featuring endogenous investment, grading, and evaluation, to study the interaction between a school’s investment decision and its choice of grading policy. When schools control information about student ability, they choose grading strategies that do not perfectly reveal student types to evaluators. However, they also invest more to increase education quality than in a benchmark case where all information about ability is observed by evaluators. The increased investment has the potential to outweigh the cost of noisy grading.

Viewed from a broad perspective, our analysis considers competing institutions that make (real) investments to improve stochastic outcomes and also exert influence over the release of information about outcome realizations. While our focus is on competition between schools, similar considerations are important in a variety of other settings. For example, a division within a firm may exert effort to develop a prototype and simultaneously control the way in which the prototype is tested prior to undertaking a development decision. Competing for promotion, a number of employees may exert effort to improve their output, while simultaneously controlling the flow of information to superiors within the firm. In finance, firms make investments and exert effort to generate profits for shareholders, but have the capability to influence the information available to investors about their performance by controlling the way in which earnings are disclosed.

In the context of education, our results have novel implications for both the policy discussion and common industry practices. We highlight a potential benefit of noisy grading—that it leads to greater investment in education quality—and show that the benefit can dominate the costs associated with information loss. This suggests that grade inflation may not be as bad for welfare as popular discussion suggests. In fact, it may lead to better outcomes than would arise if student ability is perfectly observed. By implication, the results suggest that policies or practices that convey information about graduate ability can reduce investment in the quality of education, decreasing student ability. Licensing and entrance exams provide employers
and universities with information about applicant ability. To the extent that performance on these tests is an indicator of ability, graduate school entrance exams (such as the LSAT, GRE, MCAT, and GMAT) and state licensing exams in various industries may undermine the incentives of colleges and universities to invest in quality. College entrance exams like the SAT and ACT as well as state assessment exams may have similar unintended consequences for primary and secondary education. Paradoxically, the use of such exams may reduce investment in education quality and graduate ability.

**Mathematical Appendix**

**PROOF TO LEMMA 1:**

(Adapted from Kamenica and Gentzkow 2011). Let $\Gamma$ be a random variable that satisfies the assumptions of the lemma, with density function $g(x)$ (possible mass points are represented using the Dirac Delta function). Consider two random variables $H$ and $L$ with densities $(f_H(x), f_L(x))$ given by the following:

$$f_H(x) = \frac{xg(x)}{q} \quad \text{and} \quad f_L(x) = \frac{(1 - x)g(x)}{1 - q}.$$ 

Observe that the supports of $H$ and $L$ are identical to the support of $\Gamma$. Also observe that because the expected value of $\Gamma$ is equal to $q$, both $f_H(x)$ and $f_L(x)$ are proper density functions. Consider next the posterior belief associated with realization $x$:

$$\Pr(t = h | x) = \frac{qf_H(x)}{qf_H(x) + (1 - q)f_L(x)} = \frac{xg(x)}{xg(x) + (1 - x)g(x)} = x.$$ 

Thus, the posterior belief associated with any transcript realization is equal to the realization of the transcript itself. The likelihood ratio is monotone, as the posterior belief is monotonic in the realization. The density of the transcript realization is given by

$$qf_H(x) + (1 - q)f_L(x) = g(x).$$

For the constructed grading policy, the posterior belief is equal to the realization of the transcript and the transcript has density $g(x)$. Thus, for the grading policy constructed, $\Gamma$ is the ex ante posterior belief. ■

**PROOF TO LEMMA 3:**

A strategy for player $j$ is a random variable $\Gamma_j$ with support contained in the unit interval and expectation $q_j$. Because the underlying signal structure is valid, $\Gamma_j$ has a finite number of mass points $m_k^j$, contained in set $M_j$. Let $\mu_k^j = \Pr(\Gamma_j = m_k^j)$. Ignoring mass points, random variable $\Gamma_j$ has support on a (closed) interval $I_j$. Denote $\text{support}(\Gamma_j) = I_j \cup M_j$ as $S_j$. 


Let \( W_j(x) \) represent the probability that graduate \( i \) is selected when \( i \)'s posterior belief realization is equal to \( x \): 

\[
W_j(x) = \Pr(\Gamma_j < x) + \frac{1}{2} \Pr(\Gamma_j = x).
\]

Thus, for any point inside \( x \in S_j \), this function is given by the following expression:

\[
W_j(x) = \begin{cases} 
P_j(x) & \text{if } x \in I_j \cap M_j^C \\
P_j(x) - \frac{1}{2} \mu_k^j & \text{if } x = m_k^j
\end{cases}
\]

Here we use \( X^C \) to represent the complement of set \( X \). Also note that function \( W_j(x) \) maintains a constant value in any interval that does not intersect \( S_j \). This is because \( P_j(x) \) neither increases nor decreases outside of set \( S_j \).

As is the case for \( \Gamma_j \) the support of a strategy for player \( i \), is \( S_i = I_i \cup M_i \), where \( I_i \) represents an interval and \( M_i \) represents a finite set of mass points; \( m_k^i \) represents mass point \( k \) and \( \mu_k^i \) represents the mass on point \( m_k^i \). The best response of player \( i \) to \( \Gamma_j \) is a choice of random variable \( \Gamma_i \) with generalized density \( p_i(x) \) to solve the following maximization:

\[
\max \int_0^1 p_i(x)W_j(x) \, dx
\]

subject to

\[
\int_0^1 p_i(x)x \, dx = q_i.
\]

Consider the Lagrangian for this problem:

\[
L = \int_0^1 p_i(x)(W_j(x) - \lambda_i(x - \gamma_i)) \, dx.
\]

Standard maximization principles require that the value of the integrand of the Lagrangian is the same at any value of \( x \) inside the support of \( \Gamma_i \) and no value outside of the support of \( \Gamma_i \) gives a higher value. Therefore:

\[
L_i(x) \equiv W_j(x) - \lambda_i(x - q_i)
\]

\[
x \in S_i \Rightarrow L_i(x) = v_i
\]

\[
x \not\in S_i \Rightarrow L_i(x) \leq v_i.
\]

These conditions imply several properties of best-responses.
Properties of best responses:

1. If \( \Gamma_i \) is a best response to \( \Gamma_j \), its interval support \( I_i \) is a weak subset of the interval support of \( \Gamma_j : I_i \subseteq I_j \).

Suppose \( I_i \cap I_j^c \), is nonempty. If so, it contains an interval. Outside \( I_j \), only support of \( \Gamma_j \) is a finite set of mass points. Hence, there exists a subinterval in \( I_i \cap I_j^c \) that does not intersect \( S_j \). On this interval, however, \( W_j(x) \) is constant, while \( \lambda_i(x - q_i) \) is increasing and \( L_i(\cdot) \) is nonconstant. This contradicts equation (A2).

2. For any best response \( \Gamma_i \), if \( \Gamma_j \) has a mass point on \( m_j \in M_j \), then there exists \( \epsilon \) such that \( (m_j - \epsilon, m_j) \) does not intersect \( I_i \).

Because \( W_j(x) \) jumps up at \( m_j \) but \( \lambda_i(x) \) does not, \( L_i(x) \) jumps up at \( m_j \). Hence, any value of \( x \geq m_j \) cannot give the same value of \( L(\cdot) \) as a value of \( x \in (m_j - \epsilon, m_j) \).

3. If \( \Gamma_j \) does not have a mass point on 1 and the right endpoint of \( I_j \) is strictly less than 1, then player \( i \) best response \( \Gamma_i \) does not have a mass point on 1.

If \( \Gamma_j \) does not have a mass point on 1 and the right endpoint of \( I_j \) is strictly less than 1, then for sufficiently small \( \epsilon \), \( L_i(1 - \epsilon) > L_i(1) \).

These properties have significant implications for the structure of possible equilibria.

Properties of Equilibrium

1. In equilibrium no mass point inside \([0, 1)\) can be common to both \( \Gamma_i \) and \( \Gamma_j \).

Suppose a mass point exists at \( m \) and \( \Pr(\Gamma_j = m) = \mu \). \( W_j(\cdot) \), the probability of winning for player \( i \), jumps up at \( m \) by \( \mu / 2 \). The second component of the Lagrangian, \( \lambda_i(x - q_i) \), is continuous. Hence, \( L_i(\cdot) \) jumps up at \( m \) by \( \mu / 2 \) i.e., for any \( \epsilon > 0 \), \( L_i(m + \epsilon) > L(m) \). If there exists a positive value \( \epsilon \) for which \( m + \epsilon < 1 \), then this contradicts condition (A2). Hence, only possible common mass point is \( m = 1 \).

2. In equilibrium \( I_i = I_j \).

Direct consequence of point 1 in properties of best responses.

3. In equilibrium, the smallest element of the (identical) interval support \( I \), is 0.

In equilibrium, both random variables are supported on the same interval \( I \). Suppose the smallest element of \( I \), denoted \( x \) is strictly above 0. Because
no common mass point exists in $[0, 1)$ at most one of $\Gamma_i$ and $\Gamma_j$ can have a mass point on $x$. If exactly one has a mass point on $x$, let $\Gamma_i$ be the random variable with no mass point on $x$. Besides interval $I$, $\Gamma_i$ is supported on a set of mass points. This implies that for sufficiently small $\epsilon$, no mass point exists between $x - \epsilon$ and $x$. For $\Gamma_i$, no mass point exists on $x$ and, because $x$ is the smallest element of $I$, $F_j(x) = 0$. Hence, $W_j(x) = W_j(x - \epsilon)$. However, the second component of the Lagrangian is decreasing. Thus $L(x - \epsilon) > L(x)$. This contradicts condition (A2).

(4) In equilibrium, no mass point exists in $(0, 1)$ for either player.

In equilibrium each player’s strategy has the same interval support $I = [0, r]$. By point 2 of properties of best responses, no mass point can exist in interval $(0, r]$. Otherwise, a gap must exist in the interior of $I$. However, if a player has mass point above $r$, then it must be shared with the other player. If it is not shared with the other player, then a point just below generates the same winning probability, but lower $\lambda_i(x - q_i)$ and hence a greater value of $L(\cdot)$. However, according to property of equilibrium 1, the only possible common mass point is $1$.

(5) In equilibrium, if $\Gamma_i$ has a mass point on $1$, then $\Gamma_j$ also has a mass point on $1$.

$\Gamma_i$ has a mass point on $1$, then exists an interval $(1 - \epsilon, 1)$ outside of the support of $\Gamma_j$. If no mass point on $1$ is part of $j$’s strategy, then, because the probability of $i$ winning is constant outside of $S_j$, then there exists $\epsilon$ for which $L_i(1 - \epsilon) > L_i(1)$, contradicting condition (A2).

These conditions imply that an equilibrium strategy for player $j \in \{a, b\}$ must have the following structure:

$$\Gamma_j = \begin{cases} 
\Phi_j & \text{with probability } m_j \\
1 & \text{with probability } n_j
\end{cases},$$

where $m_j + n_j = 1$ and $\Phi_j$ is a random variable with support over an interval $I = [0, r]$ and no mass points, except possibly at $0$. Note, however, that at most one equilibrium strategy can have a mass point at zero. The CDF of $\Phi_j$ is given by $F_j(x)$, where $F_j(x)$ is continuous (and differentiable), $F_j(0) \geq 0$ and $F_j(r) = 1$. In this case the win-probability for player $i$ has the following structure:

$$W_j(x) = \begin{cases} 
m_j F_j(x) & \text{if } x \in [0, r] \\
1 - \frac{n_j}{2} & \text{if } x = 1
\end{cases}.$$
The above structure allows us to simplify (A2). For $i, j \in \{a, b\}$ and $i \neq j$, condition (A2) reduces to the following:

\[(A3) \quad m_j F_j(x) - \lambda_i (x - q_i) = v_i \text{ for all } x \in [0, r]\]

\[n_i > 0 \Rightarrow 1 - \frac{n_j}{2} - \lambda_i (1 - q_i) = v_i.\]

The first part of this condition therefore implies that:

\[F_j(x) = \frac{v_i - \lambda_i q_i}{m_j} + \frac{\lambda_i}{m_j} x\]

\[F_i(x) = \frac{v_j - \lambda_j q_j}{m_i} + \frac{\lambda_j}{m_i} x.\]

It cannot be that both $(\Gamma_i, \Gamma_j)$ have a mass point on 0 (There can be no common mass points in $[0, 1]$). Thus, at most one posterior belief distribution has a mass point on 0. Let this be $\Gamma_j$.

\[F_i(0) = 0 \quad \text{and} \quad F_j(0) \geq 0 \]

\[\iff \quad \frac{v_j - \lambda_j q_j}{m_i} = 0 \quad \text{and} \quad \frac{v_i - \lambda_i q_i}{m_j} \geq 0.\]

Neither player can have a mass point at $r$. Therefore,

\[F_i(r) = F_j(r) = 1\]

\[\iff \quad \frac{v_j - \lambda_j q_j}{m_i} + \frac{\lambda_j}{m_i} r = 1 \quad \text{and} \quad \frac{v_i - \lambda_i q_i}{m_j} + \frac{\lambda_i}{m_j} r = 1.\]

In addition each strategy must satisfy the appropriate mean constraint. Given the above conditions,

\[E[\Phi_i] = \frac{\lambda_j r^2}{2m_i} \quad \text{and} \quad E[\Phi_j] = \frac{\lambda_i r^2}{2m_j}.\]

Therefore the mean constraints are

\[m_i E[\Phi_i] + n_i = q_i \iff \lambda_j \left(\frac{r^2}{2}\right) + n_i = q_i\]

\[m_j E[\Phi_j] + n_j = q_j \iff \lambda_i \left(\frac{r^2}{2}\right) + n_j = q_j.\]
Thus, every equilibrium must satisfy the following conditions (collected from above):

\[
\frac{v_j - \lambda_j q_j}{m_i} = 0 \quad \text{and} \quad \frac{v_i - \lambda_i q_i}{m_j} \geq 0
\]

\[
\frac{v_j - \lambda_j q_j}{m_i} + \frac{\lambda_j}{m_i} r = 1 \quad \text{and} \quad \frac{v_i - \lambda_i q_i}{m_j} + \frac{\lambda_i}{m_j} r = 1
\]

\[
\lambda_j \left( r^2 \right) + n_i = q_i \quad \text{and} \quad \lambda_i \left( r^2 \right) + n_j = q_j
\]

\[
m_i + n_i = 1 \quad \text{and} \quad m_j + n_j = 1.
\]

Next, observe that the equilibrium properties imply that only two equilibrium structures are possible. In the first, \( n_i = n_j = 0 \) and in the other \( n_i > 0 \) and \( n_j > 0 \) (See point 5 of equilibrium properties).

**Case I**: \( n_i = n_j = 0 \Rightarrow m_i = m_j = 1 \). In this case, system (C) reduces to the following:

\[
v_j = \lambda_j q_j \quad \text{and} \quad v_i \geq \lambda_i q_i
\]

\[
\lambda_j r = 1 \quad \text{and} \quad v_i - \lambda_i q_i + \lambda_i r = 1
\]

\[
\lambda_j \left( r^2 \right) = q_i \quad \text{and} \quad \lambda_i \left( r^2 \right) = q_j.
\]

The unique solution of this system is

\[
r = 2q_i \quad \text{and} \quad \lambda_i = \frac{q_j}{2q_i^2} \quad \text{and} \quad \lambda_j = \frac{1}{2q_i} \quad \text{and} \quad v_i = \frac{q_j - 2q_i}{2q_i} \quad \text{and} \quad v_j = \frac{q_i}{2q_i}.
\]

Finally, \( v_i - \lambda_i q_i = 1 - q_j/q_i \). In order for the required inequality, \( v_i \geq \lambda_i q_i \), to hold, it must be that \( q_j \leq q_i \). Hence, the school with greater investment, school \( A \), plays the role of school \( i \) in this construction. It is simple to check that whenever \( q_d \leq 1/2 \), the proposed random variables are admissible and valid. This is exactly the equilibrium described in the first part of Lemma 3 and this equilibrium is therefore unique.

**Case II**: \( n_i > 0, n_j > 0 \). In addition to conditions (C), we also obtain the following indifference conditions from equation A2 because of the mass points on realization one:

\[
1 - \frac{n_j}{2} - \lambda_j(1 - q_i) = v_i \quad \text{and} \quad 1 - \frac{n_i}{2} - \lambda_i(1 - q_j) = v_j.
\]
The solution of the system of equations is given by

\[ r = 2(1 - q_i) \text{ and } \lambda_i = \frac{1}{2q_j^2} \text{ and } \lambda_j = \frac{1}{2q_j} \]

and \( m_i = \frac{1}{q_i} - 1 \) and \( m_j = \frac{q_j + q_i^2 - 2q_i q_j}{q_i^2} \)

\[ n_i = 2 - \frac{1}{q_i} \text{ and } n_j = \frac{q_j}{q_i} \left(2 - \frac{1}{q_i}\right). \]

Finally, \( v_i \geq \lambda_i q_i \Rightarrow q_i \geq q_j \). Hence, school A plays the role of school i. Note that \( F_j(0)m_j = 1 - q_j/q_i \). Thus, \( \Gamma_j \) has a mass point of \( 1 - q_j/q_i \) on zero, uniform mixing from 0 to \( 2(1 - q_i) \) and mass point on one, of \( n_j \). It is simple to check that whenever \( q_a > 1/2 \), the proposed random variables are admissible and valid. This is exactly the equilibrium described in the second part of the Lemma 3 and this equilibrium is therefore unique.

To calculate school payoffs, note that \( \Gamma_b \) can always be represented in the following way:

\[
\Gamma_b = \begin{cases} 
0 & \text{with prob } 1 - \frac{q_b}{q_a} \\
\Gamma_a & \text{with prob } \frac{q_b}{q_a}
\end{cases}
\]

Hence, with probability \( q_b/q_a \) schools use an identical posterior belief distribution and they are therefore equally likely to generate the higher realization. With complementary probability, school A wins, as school B reveals its graduate to be low ability for certain.

\[
u_a = 1 - \frac{q_b}{q_a} + \frac{1}{2} \frac{q_b}{q_a} = 1 - \frac{q_b}{2q_a} \quad u_b = \frac{1}{2} \frac{q_b}{q_a} \]

PROOF OF PROPOSITION 1:

Points (i) and (ii) follow immediately from the discussion in the text and the equilibrium characterization. We prove points (iii) and (iv) by showing that whatever underlying signal structure (grading policy) school A chooses, in equilibrium school B can use a grading policy that assigns a special transcript \( F \) that reveals that the student is low ability and with complementary probability generates a transcript using school A’s grading policy (point (iii)). Thus, because school B sends an identical signal to A with some probability, but with complementary probability reveals the low state, school A’s signal is a garbling of B’s in the sense of Blackwell.

Suppose that school A utilizes an equilibrium grading policy \((H_a, L_a)\), where the density of \( H_a \) is \( h_a(x) \) and the density of \( L_a \) is \( l_a(x) \). Thus, the posterior belief about
ability at school A given transcript $x$ is $(q_a h_a(x))/(q_a h_a(x) + (1 - q_a)l_a(x))$ and the density of this posterior is $q_a h_a(x) + (1 - q_a)l_a(x)$. In equilibrium, the distribution of posterior beliefs at both schools is identical, except that school $B$ reveals a probability mass of $1 - q_b/q_a$ students to be low ability. In order to do so, school $B$ assigns an additional grade (or set of grades) that we call $F$. This grade is assigned only to low-ability students; thus observing $F$ reveals that the student is low ability. In equilibrium the probability that school $B$ assigns $F$ is $1 - q_b/q_a$. Therefore, the conditional probability that a student gets an $F$ given that he is low ability is

$$\phi = (1 - q_b/q_a)/(1 - q_b) = (q_a - q_b)/(q_a(1 - q_b)).$$

Suppose that school $B$ grading policy is identical to school $A$, except for the inclusion of the $F$ grade, assigned to low-ability students with conditional probability $\phi$. The posterior belief about $B$’s graduate if the transcript is not $F$ is given by $(q_b h_a(x))/(q_b h_a(x) + (1 - q_b)(1 - \phi)l_a(x)) = (q_a h_a(x))/(q_a h_a(x) + (1 - q_a)l_a(x))$ and the density of this posterior is $q_a h_a(x) + (1 - q_a)l_a(x)$. Thus, school $B$’s equilibrium grading policy assigns an $F$ to low-ability students with conditional probability $\phi$ and otherwise is identical to the grading policy of school $A$. ■

**PROOF OF LEMMA 4:**

First, we derive the strategic grading investment levels. With strategic grading, the payoff function for each school $i \in \{\alpha, \beta\}$ is given by

$$u_i(q_i, q_j) = \begin{cases} q_i - \frac{q_i^2}{2q_j - \rho_i^2} & \text{if } q_i \leq q_j \\ 1 - \frac{q_j}{2q_i} - \frac{q_j^2}{\rho_i^2} & \text{if } q_i > q_j \end{cases}.$$ 

This function is differentiable, continuous, and (weakly) concave in $q_i$. Therefore the unique maximum occurs wherever the derivative of the payoff function is equal to zero:

$$\frac{du_i(q_i, q_j)}{dq_i} = \begin{cases} \frac{1}{2q_j} - \frac{2q_i}{\rho_i^2} & \text{if } q_i \leq q_j \\ \frac{q_j}{2q_i} - \frac{2q_i}{\rho_i^2} & \text{if } q_i > q_j \end{cases}.$$ 

Suppose that in equilibrium $q_a \geq q_b$. Such an equilibrium is described by the following first order conditions:

$$\frac{1}{2q_a} - \frac{2q_b}{\rho_b^2} = 0 \quad \text{and} \quad \frac{q_b}{2q_a^2} - \frac{2q_a}{\rho_a^2} = 0.$$ 

Therefore we find that

$$q_a = \sqrt{\frac{\rho_a \rho_b}{2}} \quad \text{and} \quad q_b = \sqrt{\frac{\rho_a \rho_b}{2}} \sqrt{\frac{\rho_b}{\rho_a}}.$$
The ratio $q_{\beta}/q_{\alpha} = \rho_{\beta}/\rho_{\alpha}$. Hence, to be consistent with $q_{\alpha} \geq q_{\beta}$, it must be that $\rho_{\alpha} \geq \rho_{\beta}$, that is, the school with the higher resource, $\alpha$, invests more in equilibrium (thereby playing the role of school $A$). Thus,

$$q_{\alpha} = \frac{\sqrt{\rho_{\alpha}\rho_{\beta}}}{2} \quad \text{and} \quad q_{\beta} = \frac{\sqrt{\rho_{\alpha}\rho_{\beta}}}{2} \rho_{\alpha}$$

$$u_{\alpha} = \frac{\rho_{b}}{4\rho_{a}} \quad \text{and} \quad u_{\beta} = 1 - \frac{3\rho_{b}}{4\rho_{a}}.$$  

PROOF OF PROPOSITION 4:

The fully informative grading investment levels are derived in the text and the strategic grading investment levels are derived in Lemma 4. Next, we compare investments under the two grading regimes. The following inequalities define the conditions under which investment under strategic grading is higher than under fully revealing grading.

School $\alpha$: $\frac{\sqrt{\rho_{\alpha}\rho_{\beta}}}{2} \geq \frac{\rho_{\alpha}^2}{4} \iff \rho_{\beta} \geq \frac{\rho_{\alpha}^3}{4}$

School $\beta$: $\frac{\sqrt{\rho_{\alpha}\rho_{\beta}}}{2} \frac{\rho_{\beta}}{\rho_{\alpha}} \geq \frac{\rho_{\beta}^2}{4} \iff 2 \geq \sqrt{\rho_{\alpha}\rho_{\beta}}$

The last inequality is always satisfied given the assumption $\rho_{\alpha} \leq 2$ and $\rho_{\beta} \leq 2$. This establishes the two points.

PROOF OF PROPOSITION 5:

We calculate the equilibrium payoffs for the evaluator. In either case, the evaluator’s payoff is given by the following expression:

$$(A4) \quad \frac{q_{b}}{q_{a}} E[\Gamma_{a}^{(2)}] + \left(1 - \frac{q_{b}}{q_{a}}\right)q_{a}$$

Here, $\Gamma_{a}^{(2)}$ represents the maximum order statistic from two draws of random variable $\Gamma_{a}$. If school $b$ does not reveal the student to be low ability for sure, then it uses the school $A$ grading policy. In this case the evaluator payoff is the maximum draw. If, however, school $b$ reveals its graduate to be low ability, the evaluator only receives the expected quality of a graduate from school $A$, which is $q_{a}$.

Case I: $q_{a} \leq (1/2)$. In this case, $\Gamma_{a} = U[0, 2q_{a}]$ and therefore, $E[\Gamma_{a}^{(2)}] = (4/3)q_{a}$. Evaluating (A4) gives expected evaluator payoffs

$$\frac{q_{b}}{q_{a}} \left(\frac{4}{3} q_{a}\right) + \left(1 - \frac{q_{b}}{q_{a}}\right)q_{a} = q_{a} + \frac{1}{3}q_{b}.$$
Case II: $q_a > (1/2)$. Here, $\Gamma_a = U[0, 2(1 - q_a)]$ with probability $1/q_a - 1$ and $\Gamma_a = 1$ otherwise. Thus, evaluating $E[\Gamma_a^{(2)}]$ gives

$$E[\Gamma_a^{(2)}] = 1 - \left(\frac{1}{q_a} - 1\right)^2 + \left(\frac{1}{q_a} - 1\right)^2 \left(\frac{4}{3}\right)(1 - q_a).$$

Substituting and simplifying gives

$$E[\Gamma_a^{(2)}] = \frac{1 - 6q_a + 12q_a^2 - 4q_a^3}{3q_a^2}.$$

Combining this with (A4) gives

$$u_e = \frac{3q_a^4 - 7q_a^3q_b + 12q_a^2q_b - 6q_aq_b + q_b}{3q_a^3}.$$

We find the evaluator payoff by substituting the school effort levels into the evaluator payoff function. The two cases arise because $\rho_{\alpha}\rho_{\beta} \leq 1$ implies $q_{\alpha} \leq (1/2)$, which influences the nature of the equilibrium that arises in the second stage.

Here, we calculate the evaluator welfare under fully-revealing grading. Under fully revealing grading, the evaluator’s payoff is the probability that at least one graduate has high ability:

$$u_e = \frac{\rho_i^2 + \rho_j^2}{4} - \frac{\rho_i^2 \rho_j^2}{16}.$$

Next, we compare the evaluator’s payoff under fully revealing and strategic grading. The following inequalities define the region of $(\rho_{\alpha}, \rho_{\beta})$ for which the evaluator payoff is higher with strategic grading.

$$\left(\frac{3\rho_{\alpha} + \rho_{\beta}\sqrt{\rho_{\alpha}\rho_{\beta}}}{6}\right) - \left(1 - \left(1 - \frac{\rho_i^2}{4}\right)\left(1 - \frac{\rho_j^2}{4}\right)\right) \geq 0 \quad \text{if } \rho_{\alpha}\rho_{\beta} \leq 1$$

$$\left(\frac{3\rho_{\alpha}^2 - 7\rho_{\alpha}\rho_{\beta} - 24\sqrt{\rho_{\alpha}\rho_{\beta}} + 24\rho_{\alpha}\rho_{\beta} + 8}{6\rho_{\alpha}^2}\right) - \left(1 - \left(1 - \frac{\rho_i^2}{4}\right)\left(1 - \frac{\rho_j^2}{4}\right)\right) \geq 0 \quad \text{if } \rho_{\alpha}\rho_{\beta} > 1$$

This function is continuous. We establish that these inequalities are satisfied along the diagonal $\rho_{\alpha} = \rho_{\beta}$ and therefore hold in a region around the diagonal. Let $\rho_{\alpha} = \rho_{\beta} = \rho$. These inequalities become:

$$\frac{1}{16}\rho^4 - \frac{1}{2}\rho^2 + \frac{2}{3}\rho \geq 0 \quad \text{if } \rho \leq 1$$

$$\frac{1}{48}\rho^2(2 - \rho)^2((2 - \rho)^4 + 2\rho^2(\rho - \sqrt{31} + 5)(\rho + \sqrt{31} + 5) \geq 0 \quad \text{if } 1 \leq \rho \leq 2.$$
The first inequality is satisfied because, \((2/3) \rho - (1/2) \rho^2 \geq 0\) for all \(0 \leq \rho \leq (4/3)\). The second inequality is also satisfied because in the region of interest, as all terms in the sum are positive. Thus, around the diagonal, the evaluator payoff is higher under strategic grading than under fully informative grading. In addition, plotting these regions using a software package capable of implicit plots shows that the inequalities hold in an arc-shaped region around the diagonal. ■

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