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History-Bound Reelections[†]

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We introduce history-bound reelections. In their simple form, they consist in a "score-replication rule." Under such a rule, an incumbent has to match the highest vote share he or she has obtained in any previous election in order to be reelected. We develop a simple three-period model to examine score-replication rules. We show that suitable variants of such rules can improve welfare, as they reduce the tendency of reelected incumbents to indulge in their own preferences, and they ensure that able officeholders are reelected. Candidates might offer their own score-replication rule in campaigns. We outline how political competition may be affected by such new forms of elections. (*JEL* D72, D83)

The lack of competition in election races is a long-acknowledged issue in democracy research. It has now become a focus of academic and public debate in the United States. Frequently, this absence of genuine competition is due to the fact that one of the candidates for office is an incumbent, who appears to be reelected quite easily. In the US Congress, the number of reelected incumbents has been rising regularly. In 2014, it was above 90 percent in the US House of Representatives.[1] A rich literature has developed various explanations for this lack of competition, such as the redistricting hypothesis, the partisan polarization hypothesis, and the incumbency hypothesis. While we do not address redistricting in this paper, partisan polarization and incumbency advantages are the focus of our analysis.[2]

While Downs' (1957) famous median voter result is a consequence of competitive elections, noncompetitive elections can lead to policy polarization. Moreover, strong incumbency advantages may increase shirking, reduce the effort an officeholder is willing to invest in important tasks, and may foster the reelection of less able officeholders. In such cases, incumbency advantages are welfare-reducing. In some cases, it is difficult to influence the sources of such incumbency advantages directly,

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[1] See *Economist* (2014), Buchler (2007), Mayhew (1974), and Jacobson (1987, 2004).

[2] It is difficult to assess precisely why the incumbency advantage has grown. The incumbents may have greater financial advantages today because they are better able to deter challengers from candidacy, they have closer ties to constituencies, or they benefit from gerrymandering. All of these factors may be (partial) explanations for the development observable in the United States (see, e.g., Buchler 2007).

as these are given. In the United States, for instance, the benefits awarded to members of Congress comprise heightened visibility and perks such as franking privileges, the right to send print material to their constituents at the taxpayers' expense (Glassman 2007). Such a financial advantage might lose some of its importance because of electronic messaging. However, the broadcasting of debates through C-SPAN television, amplified by the broadcasting of excerpts and reruns on local news, still offers officeholders a platform that increases their notoriety—free of charge. It would thus be useful to have an election rule that makes elections more competitive. At the same time, such a rule should be able to preserve those effects of incumbency advantages that are beneficial to society.[3] To achieve this goal, we suggest tying reelection to the incumbents' electoral history.

Let us consider a two-candidate election contest for an office in the executive or legislative branch. In its simplest form, history-bound reelections would involve a special rule for incumbents applying for reelection, worded as follows.

Score-Replication Rule for Elective Offices:

(*i*) *If a candidate is not the incumbent, a simple majority of votes will suffice for election.*

(*ii*) *If a candidate is the incumbent, he must attain the highest percentage of votes he has ever obtained for this same office.*

(*iii*) *If the incumbent fails to obtain his highest past percentage, he will not be elected.*

Hence, under the score-replication rule, the incumbent's best previously attained percentage acts as a threshold that must not be undermatched for reelection. Accordingly, the incumbent has to overcome a hurdle that is potentially higher than for a first-time candidate. To achieve this, all voters who initially voted for him/her must vote for his/her reelection or be replaced by at least an equivalent number of new voters. Thus, an officeholder will have to perform as well as possible to keep the percentage of votes constant or to raise it above the best previous election result. Such a special requirement for incumbents should reintroduce competition into elections by reducing the degree of political polarization that incumbents having a large advantage might pursue.[4] Moreover, it only allows incumbents with sufficient ability levels to be reelected.

Several variants of this rule are conceivable. For example, the level of votes to be attained could be lowered by a certain percentage compared to the best past score. One might also disregard the first election outcome when the reelection hurdle for

[3] Another way to render elections more competitive might be to delineate more competitive districts to prevent gerrymandering. This would not, however, reduce incumbency advantages.

[4] Abramowitz, Alexander, and Gunning (2006) have developed the "polarization hypothesis" as an explanation for weak electoral competition.

the incumbent is calculated. Another option would be to have the incumbent attain the average percentage of all votes earned in previous elections.

If an incumbent fails to replicate—or surpass—his/her best past result (possibly lowered by some margin), the situation could be dealt with in two ways. One way would be to declare the challenger the automatic winner. However, this new officeholder may have received quite a small percentage of votes, which would cast doubt on his legitimacy. Alternatively, one could declare a run-off between the challenger and a new candidate to ensure that the new officeholder is supported by at least 50 percent of the voters.

It is important to clarify the contribution of this paper. The source of the voter's problem is that reelection is a crude tool to exercise control on policy choices. If voters could use a finely tuned mechanism—e.g., a complete contract—the problem could be alleviated. Yet, in this paper, we look for a more modest improvement of the election mechanism that remains close enough to the majority rule, and thus could be used in practice and offer interesting properties. Hence, there is a trade-off between the distance to reality and the potential and implications of a new election rule. It appears that score-replication rules are interesting enough and feasible, so that this trade-off is worth exploring.

Model and Results.—We use formal theory to define and explore score-replication rules. In particular, we use a simple three-period model to define and study history-bound reelections. Officeholders undertake public projects that benefit all citizens and where the output depends on their ability. They also choose an ideological (or redistributive) policy over which voters usually disagree. Both the ability and the preferred ideological policy of the officeholder are private information.

Ability can be inferred by voters from public project provision. The ideological policy choice can also provide information about the officeholder's preferences depending on whether he chooses the median voter's ideal policy (no information revelation) or indulges in his own preferences (partial or even complete information revelation).

An elected politician whose ability turns out to surpass some given threshold will be able to secure reelection. He will subsequently deviate just so much from the median voter's ideal policy toward his own ideal policy that a majority will still prefer him over a challenger with unknown ability and ideological preference. Thus, the officeholder exploits his incumbency advantage regarding ability to implement more polarized ideological positions.

We characterize the equilibria of such political election contests in a simple three-period model. We then show that score-replication rules reduce the ideological bias of officeholders while the expected ability of politicians in office is not affected. As a result, score-replication rules yield higher expected welfare than standard elections.

Subsequently we discuss how extensions and further characteristics of political competition—such as welfare-reducing incumbency advantages and initial heterogeneities of candidates—strengthen the result. In cases when the score-replication rule might introduce negative welfare effects, we introduce weaker forms of the rule (e.g., one could disregard the first election result, or the percentage of votes an officeholder must attain could be reduced by some margin compared to the percentage of votes achieved in previous elections).

We conclude with some general remarks on how political competition may be affected by such new forms of elections. This opens a new field of research and might trigger a fresh approach to democracy.

The paper is organized as follows. In the next section, we introduce the model and the equilibrium concept. In Section II, we analyze and characterize the outcomes with standard elections. In Section III, we introduce the score-replication rule and examine the policy choices and reelection outcomes in this environment. In Section IV, we establish the welfare result. In Section V, we explore various extensions and introduce weaker forms of the score-replication rule. In Section VI, we conclude with a more general discussion of history-bound reelections.

I. The Model

We consider three periods, denoted by $t = 1,2,3$. The electorate consists of a continuum of voters indexed by $i \in [0,1]$. There are two sets of candidates: left-wing (denoted by L) or right-wing (denoted by R). Candidates and officeholders are denoted by k, k', or k''. In each period, one left-wing and one right-wing candidate compete for office. The winner of the first election is the incumbent in the second election and faces a challenger from the other side of the political spectrum. The winner of the second election will again face a challenger with opposing political ideology in the third election.

A. *Policies and Utilities*

In each period, the officeholder generates or affects utility for citizens with policy choices in two dimensions:

- Ideological Policy: I
  An elected politician selects an ideological policy I in the policy space $[0,1]$. In a given period t, the choice of policy I by officeholder k will be denoted by i_{kt}. Voters are ordered according to their ideal points, with i being the ideal point of voter i. Voter i derives utility $-|i_{kt} - i|$ from the policy i_{kt}.[5] We assume that ideal points of voters and thus voters themselves are uniformly distributed in $[0,1]$.

- Public Project: P
  The incumbent undertakes a public project in each period. We denote the level of its provision in period t by g_t. The level of g_t is assumed to depend only on the incumbent's ability—denoted by a_k—and is given for simplicity by[6]

[5] We note that the utility has been normalized to zero when the policy choice is equal to the most preferred policy of a voter.

[6] In an extension in Section VIA, g_t will differ from a_k.

(1) $$g_t = a_k.$$

We assume that the ability a_k of an officeholder is randomly drawn from a uniform distribution on $[-A, A]$ with $A > 0$.[7]

We note that policy quality is solely a function of type and not effort. This removes incentives to strategically reduce quality by choosing low effort in order to avoid facing high future election hurdles. The utility of voter i in period t is denoted by v_{it} and given by

(2) $$v_{it} = \lambda a_k - |i_{kt} - i|,$$

where $\lambda > 0$ is the weight voters put on utilities from public project provision versus utilities from ideological policies. Hence, all voters have the same preferences regarding the public project P, while they have differing preferences over the ideological policy I, which in period t is given by i_{kt}. The expected lifetime utility of voter i is denoted by V_i. To simplify the analysis, we assume that voters (and politicians) have a discount factor of one.[8]

The politicians derive utility from two sources:

- Policies
  Politicians are assumed to derive the same benefits from policies I and P as voters who share their ideal points. The right-wing candidate's most preferred point with regard to I is denoted by μ_R, while that of a left-wing candidate is denoted by μ_L. We assume that μ_R and μ_L are private information. From the voters' perspective, μ_L and μ_R lie in $[\hat{\mu}_L, 1/2)$ and $(1/2, \hat{\mu}_R]$, respectively, where $0 < \hat{\mu}_L < 1/2$ and $1 > \hat{\mu}_R > 1/2$ denote the most extreme possible positions, which are common knowledge. From the voters' perspective, μ_L and μ_R are uniformly distributed in $[\hat{\mu}_L, 1/2]$ and $[1/2, \hat{\mu}_R]$, respectively, and $\hat{\mu}_L = 1 - \hat{\mu}_R$. Hence, the most extreme positions $\hat{\mu}_R$ and $\hat{\mu}_L$ are located symmetrically around the median. Left-wing candidates choose ideological positions in $[\hat{\mu}_L, 1/2]$, right-wing candidates positions in $[1/2, \hat{\mu}_R]$.[9]

- Officeholding
  The incumbent derives private benefits from holding office not only in the form of a salary, but also in the form of non-monetary benefits such as prestige or the satisfaction of being in power. These benefits are denoted by b $(b \geq 0)$.

[7] Output may indeed be negative, for instance, when officeholders start unnecessary conflicts. Of course, one can avoid output being negative by adding a constant and set $g_t = a_k + c$ with $c > A$. However, for ease of exposition, we can neglect such a constant, as it would not affect our results.

[8] The extension to discount factors lower than 1 is straightforward and Theorem 1 in Section IV holds for all discount factors equal to or lower than 1.

[9] We note that this is already an equilibrium argument. However, it is intuitive that a left-wing officeholder never has an incentive to choose an ideological position outside $[\hat{\mu}_L, 1/2]$. This can be verified for all settings considered in this paper.

The utility of a politician k in period t who is in power in this period is

$$v_k = \lambda a_k - |i_{kt} - \mu_k| + b. \tag{3}$$

An officeholder derives utility from both policy choices and holding office. The expected lifetime utility of politician k is denoted by V_k and naturally depends on whether he is in office in particular periods or not. If a politician is not in office, he derives utility as a citizen with ideal point μ_L and μ_R, respectively.

B. *Information and the Equilibrium Concept*

We assume that candidates cannot commit to an ideological policy I during campaigns. Hence, once in office, a politician chooses an ideological position that maximizes his expected lifetime utility at that time. Officeholders only learn their ability in the first term, which will manifest itself in the level of output from the public project. We assume that voters observe this output and can thus directly infer the ability of first-time officeholders. As mentioned before, the most preferred ideological position of a candidate or officeholder is private information. It may be either concealed or (partially) revealed over the course of the political game.

The overall game and the parameters, such as b and λ, and the candidates' most extreme ideological positions, $\hat{\mu}_R$ and $\hat{\mu}_L$, are common knowledge. Moreover, we assume that $\lambda A \geq 1/2$, which implies that an officeholder with high ability can generate utility for the citizens from the public project to overcompensate extreme policy choices compared to an officeholder with average ability, who will choose the median position.

The political process is governed by the simple-majority rule in the first period. Later, the score-replication rule will be applied. As a benchmark, we also will consider the case when the standard simple-majority rule is applied in all periods. Finally, we will assume that the benefits from holding office, b, are so large that candidates do not want to lower their reelection chances by choosing more ideological policies. A lower bound will be given in Proposition 6.

C. *The Overall Game*

We summarize the course of events for standard elections in the overall game with Figure 1. We seek perfect Bayesian Nash Equilibria in pure strategies of the game. Expectations are taken about the appropriate probability measure. We make one additional assumption. In the main body of the paper, we assume that citizens vote sincerely in each election (i.e., they vote for the candidate competing for office who will generate higher expected utility).[10] Later, in Section V, we will examine

[10] We assume that all voters in the continuum of citizens vote sincerely to replicate the voting behavior in a large but finite population. We also assume that each citizen casts his/her vote and there are no abstentions; a voter who is indifferent between the two candidates will select each candidate with probability $1/2$. Moreover, since the overall game is extremely complex and involves many subtleties, we limit ourselves to some of the main insights that can be derived without displaying the entire solution of the game.

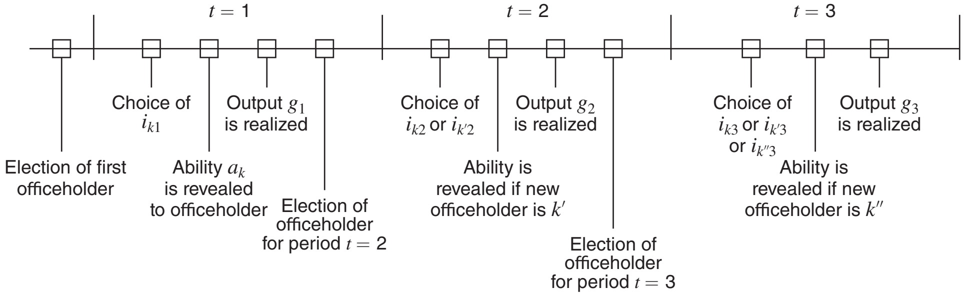

Figure 1. Timeline

strategic voting. To simplify the language, we simply refer to "equilibria" when we analyze the entire game or subgames thereof.

D. *Welfare*

We focus on expected welfare of the median voter, evaluated at the beginning of the election process. Because voters and $\hat{\mu}_R$ and $\hat{\mu}_L$ are distributed symmetrically around the median, this criterion is equivalent to maximizing expected aggregate utility of all voters.

II. Standard Elections

We first examine the equilibrium outcomes under standard election rules. This will serve as a benchmark for our analysis. With standard elections, a candidate wins if he obtains more than 50 percent of the votes. In a tie of two new candidates, each candidate wins and assumes office with probability $1/2$.[11] Because preferences of voters are single-peaked and all voters hold the same beliefs, the median voter theorem holds in every period (i.e., a candidate is reelected if the median voter $i = 1/2$ prefers him over the challenger). In our analysis, we proceed backwards, beginning with the last two periods.

A. *The Second and Third Period*

Preliminaries.—The best policy for an officeholder in the last period is to choose his preferred ideological position, independent of whether the officeholder is in his first term and does not know his ability, or whether he was incumbent and his ability is common knowledge. This is due to the fact that there are no reelection concerns. That is, in the third period, any left-wing officeholder k chooses $i_{k3} = \mu_L$, and any right-wing officeholder k' chooses $i_{k'3} = \mu_R$.

[11] If one candidate is an incumbent, we assume that he is reelected when he obtains at least 50 percent of the votes and thus when the median voter is at least as well off with his policies than with those of a challenger. This is simply for reasons of convenience and to avoid working with a discretized strategy space.

We next turn to the second period. We first focus on the situation when a left-wing officeholder k in period $t = 2$ is in his second term. Suppose that such an officeholder has chosen i_{k2} as his ideological policy, and that his ability has turned out to be a specific value a_k. We denote by $\mu_{L2}^e(i_{k2})$ the expected ideal point of the left-wing officeholder, expected by voters upon observing the choice i_{k2}, $i_{k2} \in [\hat{\mu}_L, 1/2]$.[12] For a particular equilibrium constellation, this belief has to be specified for policy choices in and out of equilibrium and is described by the function $\mu_{L2}^e(i_{k2})$. We note that $\mu_{L2}^e(i_{k2}) \leq 1/2$.

Beliefs in the Second Period.—We next assume that the belief function $\mu_{L2}^e(i_{k2})$ does not depend on the first-period policy choice. That is, we examine a continuation equilibrium, given a history without signaling regarding the ideological position in the first period. Later, we show that such a history is indeed part of the equilibrium play. The reason is that officeholders only learn their ability after they have chosen their ideological position in the first period and thus would only lower their reelection chance by signaling a more extreme position than the prior expectation of voters in the first period.

We next show a property that is central for the determination of the equilibria.

PROPOSITION 1: *$\mu_{L2}^e(i_{k2})$ is strictly increasing in i_{k2} on $[\hat{\mu}_L, 1/2]$ for values i_{k2} that are chosen in equilibrium.*

The proof is given in the Appendix. The preceding result implies that the reelection prospects are better, or at least not worse, if an officeholder chooses a position closer to the median voter's ideal position in equilibrium.

We impose a refinement on out-of-equilibrium beliefs.

REFINEMENT 1: *The belief function $\mu_{L2}^e(i_{k2})$ is also non-decreasing out-of-equilibrium.*

There are at least three rationales for this refinement. First, the refinement would be an equilibrium outcome if we added some noise to the game. Suppose that there is an arbitrarily small probability ϵ $(\epsilon > 0)$ that the officeholder is myopic and is only interested in the utility he can achieve in the current period. Such officeholders would choose their preferred ideological position as a strictly dominant strategy. This would pin down beliefs on the entire interval $[\hat{\mu}_L, 1/2]$ and together with Proposition 1, it would ensure that $\mu_{L2}^e(i_{k2})$ is non-decreasing. Second, the property can be justified by passive beliefs of voters when they assume that a deviation from an equilibrium choice is equally likely for officeholders with different ideological orientations. Third, the same conclusion would arise if voters assumed that deviations to more polar positions are more likely for candidates for whom such positions are ideal or less extreme than their preferred position. This last justification means that the deviator is believed to be of the type that has most to gain from

[12] It will be sufficient to define beliefs of left-wing (right-wing) candidates on $[\hat{\mu}_L, 1/2]$ ($[1/2, \hat{\mu}_R]$).

the deviation. This is an application of classic refinements from the game theory literature; in particular, of the divinity criterion of Banks and Sobel (1987) (see Fudenberg and Tirole 1991 for an overview).

Proposition 1 and Refinement 1 imply that ideologically moderate officeholders also choose more moderate ideological positions or that they pool at the same policy as more extreme officeholders. The reason is that reelection chances cannot increase by taking less moderate ideological positions, as voters' beliefs about an officeholder's ideological preferences—and thus about future ideological policy choices—become less moderate. This is formally proven in Lemma 2 in the Appendix.

Policy Choices of an Incumbent.—After these preparations, we can determine the equilibrium policy choices. For this purpose, we will focus on existence and uniqueness. Uniqueness only involves policy choices and beliefs on the equilibrium path and expected utilities. Out-of-equilibrium beliefs are not determinate, but have to satisfy certain conditions. We focus on the case where officeholders have ability $a_k \geq 0$, as it turns out that officeholders with ability below average will be deselected.[13]

PROPOSITION 2: *Suppose that a left-wing officeholder k has been reelected for the second term and has (revealed) ability $a_k \geq 0$ with $2\lambda a_k < (1/2) - \hat{\mu}_L$. There exists, then, a unique equilibrium policy choice:*

$$(4) \qquad i_{k2} = \begin{cases} \frac{1}{2} - 2\lambda a_k & \text{for } \hat{\mu}_L \leq \mu_L \leq \frac{1}{2} - 2\lambda a_k \\ \mu_L & \text{for } \frac{1}{2} - 2\lambda a_k < \mu_L \leq \frac{1}{2} \end{cases},$$

and with this choice, the officeholder k will be reelected for the third term. The beliefs of voters are given by

$$(5) \qquad \mu^e_{L2}(i_{k2}) = \begin{cases} i_{k2} & \text{if } \frac{1}{2} - 2\lambda a_k < i_{k2} \leq \frac{1}{2} \\ \frac{1 - 4\lambda a_k + 2\hat{\mu}_L}{4} & \text{if } i_{k2} = \frac{1}{2} - 2\lambda a_k \\ \frac{i_{k2} + \hat{\mu}_L}{2} & \text{if } i_{k2} < \frac{1}{2} - 2\lambda a_k \end{cases}.$$

The proof of Proposition 2 is given in the Appendix. Proposition 2 indicates that an officeholder with above average ability has an incumbency advantage, since he can secure reelection when competing against a challenger with expected ability equal to zero. While this incumbency advantage is beneficial for voters in subsequent periods,

[13] If an officeholder with known ability $a_k < 0$ is in office in the second term, the game-theoretic considerations are subtle and complex. It is not possible that some (more moderate) officeholders with $a_k < 0$ are reelected while others with the same ability (and a more extreme ideal policy) are deselected, since the reelection probability for all types has to be the same. Hence, neither the choice $i_{k2} = \mu_L$ nor $i_{k2} = 1/2$ can constitute a general best response. As a consequence, officeholders will play some (mixed) strategies (details are available upon request). However, these out-of-equilibrium considerations do not impact the equilibrium choices described in Proposition 2 and in Proposition 4.

the officeholder partly indulges in his ideological preferences. He just deviates from the median voter position by so much that the inferred ideological orientation captured by $\mu_{L2}^e(i_{k2})$ and the associated loss of attractiveness for the median voter do not outweigh the ability advantage if this ability advantage is moderate. If his ideal point is sufficiently moderate and/or his ability is high, he will choose his ideal ideological policy. We note that out-of-equilibrium beliefs are not unique. The formulation in Proposition 2 reflects the assumption that any accidental choice $i_{k2} < (1/2) - 2\lambda a_k$ would stem from officeholders with $\mu_L \leq i_{k2}$ and that such accidental choices are equally probable among those candidates.

An immediate consequence of the considerations in Proposition 2 is that reelection concerns do not constrain the policy choice if ability is sufficiently high $(2\lambda a_k \geq (1/2) - \hat{\mu}_L)$.

Policy Choices of a Freshman.—We finally address the policy choice in the second period when an officeholder is in the first term. We obtain the following result.

PROPOSITION 3: *Suppose a left-wing officeholder k is in the first term in the second period. There exists, then, a unique equilibrium in which he chooses the median voter's ideal point* $i_{k2} = 1/2$. *The voters' beliefs about the ideological preferences of the officeholder, denoted by* $\mu_{L1}^e(i_{k2})$, *are given by*

$$\mu_{L1}^e(i_{k2}) = \begin{cases} \frac{\frac{1}{2} + \hat{\mu}_L}{2} & \text{if } i_{k2} = \frac{1}{2} \\ \frac{i_{k2} + \hat{\mu}_L}{2} & \text{if } i_{k2} < \frac{1}{2} \end{cases}. \tag{6}$$

The proof of Proposition 3 is given in the Appendix. The intuition for Proposition 3 runs as follows. A new officeholder does not know his ability yet. Hence, to maximize reelection chances, he chooses the median voter's preferred ideological policy, as this maximizes the belief that the officeholder is moderate. As all officeholders pool at $i_{k2} = 1/2$, the equilibrium beliefs are $\mu_{L1}^e(1/2) = ((1/2) + \hat{\mu}_L)/2$. Deviation to a choice $i_{k2} < 1/2$ would make it clear that the ideal ideological choice of the officeholder is not in the interval $(i_{k2}, 1/2)$ and thus the expected preferred ideological choice would decline. We note that the same considerations as for Proposition 2 have been used to describe the out-of-equilibrium beliefs.

B. *The First Period*

We next turn to the first period. We make analogous assumptions regarding the voters' beliefs. Let $\mu_{L1}^e(i_{k1})$ and $\mu_{R1}^e(i_{k1})$ denote the expected ideal point of the left-wing and right-wing officeholder respectively, from the point of view of the electorate upon observing the first-period ideological policy choice i_{k1}.

We obtain the following result.

PROPOSITION 4: *Suppose that the continuation equilibrium as described in Section IIIC is played. There exists, then, a unique equilibrium in the overall* *game in which left-wing and right-wing candidates have a chance of* $1/2$ *of being elected in the first period. Once elected, a left-wing or right-wing officeholder chooses* $i_{k1} = 1/2$. *The officeholder will be reelected if and only if*

$$a_k \geq \hat{a} := \frac{1}{2}\sqrt{\frac{A}{2\lambda}\left(\frac{1}{2} - \hat{\mu}_L\right)}. \tag{7}$$

The voters' beliefs are

$$\mu_{L1}^{e}(i_{k1}) = \begin{cases} \frac{\frac{1}{2} + \hat{\mu}_L}{2} & \text{if } i_{k1} = \frac{1}{2} \\ \frac{i_{k1} + \hat{\mu}_L}{2} & \text{if } i_{k1} < \frac{1}{2} \end{cases}. \tag{8}$$

The proof of Proposition 4 is given in the Appendix.

The equilibria and out-of-equilibria beliefs can be understood as follows. Suppose that a left-wing candidate with ideal point $\mu_L < 1/2$ has been elected. Selecting the equilibrium policy $i_{k1} = 1/2$ yields a belief $\big((1/2) + \hat{\mu}_L\big)/2$ about his policy preference, since all incumbents pool at $i_{k1} = 1/2$. By selecting a policy i_{k1} with $\mu_L \leq i_{k1} < 1/2$, the officeholder would gain in period 1, but this attempt would signal that the incumbent is a more extreme politician and the voters' assessment about his policy preference would shift to $\big(i_{k1} + \hat{\mu}_L\big)/2$. This would lower the probability of winning in the next election. If the gains from holding office are not too low, the resulting change in beliefs (and the associated lower chances to win reelection) outweighs the benefits of any incremental policy move toward the left. This is also true for the case when the officeholder jumps all the way to his ideal point, which would generate beliefs $\big(\mu_L + \hat{\mu}_L\big)/2$ regarding his ideological position and would lower reelection chances maximally.

C. *Complete Equilibria Description and Consequences*

To complete the description of all paths the election process can take, we have to specify what happens when an officeholder's ability is below the ability threshold that secures reelection (i.e., $\hat{a}$ in the first period and $a = 0$ in the second period). If the officeholder stands for reelection and is deselected, the challenger from the other side of the political spectrum is elected with certainty. If the current officeholder steps down, a new candidate with the same ideological orientation has a chance of 50 percent to keep the outcome on his preferred side of the political spectrum. Because this is in the interest of the officeholder who will be deselected with certainty, we obtain the following result.

COROLLARY 1:

(*i*) *At the end of period* 1, *an officeholder with revealed ability* $a_k < \hat{a}$ *will not stand for reelection*.

(*ii*) *At the end of period* 2, *an officeholder with revealed ability* $a_k < 0$ *will not stand for reelection*.

We note that for the median voter (and aggregate welfare), it is irrelevant whether an incumbent of insufficient ability stands for reelection or not, because the median voter is indifferent as to whether a new candidate is left-wing or right-wing.[14]

To sum up, the equilibrium developed in the preceding propositions indicates that officeholders in their second term start to use their potential ability advantage in order to indulge in their own ideological preferences. The potential benefit for voters from officeholders with higher ability will thus be reduced or even completely eliminated.

For later use, we observe that officeholders with higher ability are strictly more attractive for the median voter than candidates with lower ability and the same ideological preferences.

COROLLARY 2: *Compare two possible left-wing, second-term officeholders k and k' for period* 2, *with unrevealed ideal points $\mu_L = \mu_L'$ and revealed non-negative abilities $a_k > a_{k'} \geq 0$. From the perspective of the end of period* 1, *when the ability of the incumbent has been revealed*, *the median voter's expected utility from the officeholder k is weakly higher than the one from officeholder k'.*

The proof of Corollary 2 is given in the Appendix. Corollary 2 indicates that the median voter weakly prefers officeholders with higher ability, despite the fact that such officeholders may move further away from the median voter's ideal policy in the second term than officeholders with low ability. Thus, officeholders with higher ability will have the same or a higher chance of being reelected.

III. Score-Replication

A. *The Modified Election Rule*

We next introduce the new election rule called "score-replication rule," henceforth called SR-rule. We use it in the strongest form, as follows.

DEFINITION 1 (SR-rule): *If an officeholder is in his τth term* ($\tau \geq 1$), *he must achieve a vote share of*

$$s_\tau \geq \max_{1 \leq \tau' \leq \tau} \{s_{\tau'-1}\} \tag{9}$$

in order to be reelected for his $\tau + 1$th term, *where $s_{\tau'-1}$ denotes the realized vote share in the election for his τ'th term.*

To complete the election procedure with SR-rule, one has to specify what happens if an incumbent is deselected although he has obtained more than 50 percent of the votes. There are two possibilities. First, the challenger is automatically elected. Alternatively, to ensure that the new officeholder is supported by

[14] This is different from the score-replication rule explored in the next section.

at least 50 percent of the votes, one could select the new officeholder by a run-off between the original challenger and a new candidate.[15] In the model we consider in this paper, both versions yield the same welfare.[16]

In one of the extensions, we explore a variant of the SR-rule that prevents the first election result from impacting future outcomes. This variant is called "reelection score-replication rule" (RSR-rule). It requires that the officeholder only needs to replicate vote shares received in previous reelection bids.

B. *Outcomes*

We now investigate how the SR-rule affects election outcomes. We start from the observation that many results from standard elections carry over to elections with the SR-rule.

LEMMA 1: *With the SR-rule, Lemma* 2 *and Proposition* 3 *continue to hold with regard to the policy choices of officeholders. Moreover, officeholders choose* $i_{k1} = 1/2$ *in the first period.*

Lemma 1 holds because the calculus for officeholders in the last period or in their first term does not change. In particular, a candidate in his first term maximizes his reelection chance by choosing the median voter's preferred ideological policy.

The SR-rule, however, affects the behavior of second-term officeholders and thus the results of Proposition 2. As a consequence, voting decisions at the end of the first period and the critical ability $\hat{a}$ in Proposition 4 may also change.

We start the analysis with the observation that the SR-rule introduces a feedback effect from the election outcome of the first period to policies in the second period (if the officeholder is reelected) and back to the incentives of voters to support the reelection bid of the officeholder at the end of the first period. We address this feedback effect in two steps. In the first step we explore a constellation that subsequently turns out to be the building block when we characterize the entire equilibrium.

PROPOSITION 5: *Suppose that a left-wing officeholder k with ability* $a_k \geq 0$ *has been reelected at the end of period* 1 *for a second term with a vote share* s_1 $(s_1 \geq 1/2)$ *and the most preferred ideological position of officeholders has not been revealed. There exists, then, a unique equilibrium continuation with the properties that policy choices in period* 2 *are more moderate than or equal to the policy choices under regular elections, and that the ability threshold below which officeholders are deselected is equal to or higher than the one with standard elections.*

[15] The party whose officeholder has been deselected chooses a new candidate, and we assume that he has the same characteristics regarding the distribution of ability and possible ideological positions as specified in the model for the original candidate from the other side of the political spectrum.

[16] The reason is as follows. Since the ability and the ideological position of the challenger are private information, the run-off takes place between the challenger and the new candidate as if the challenger had been drawn anew. Hence, there are no incentives for tactical voting in the election when the run-off solution is used.

A long version of Proposition 5, with all detailed policy choices, beliefs, and election decisions, is given as Proposition 10 in the Appendix. It is useful to illustrate the reelection decisions of voters at the end of period 1 and the behavior of incumbents in the second term in relation to the revealed ability.

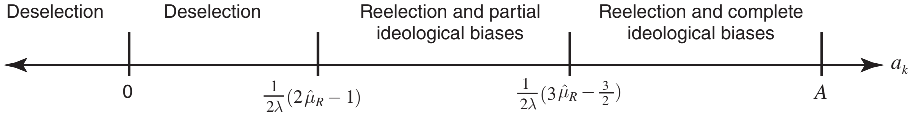

Figure 2. Behavior of Incumbent In Period 2

After these preparations, we now analyze the election outcome at the end of period 1 and the behavior of the officeholder in this period.

PROPOSITION 6: *There exist unique equilibrium choices in the first period in which both candidates choose* $i_{k1} = 1/2$ *and have a chance of 50 percent of being elected. An officeholder will be reelected if and only if his ability is above some threshold* $\hat{a}^{SR} > 0$. *The voters' beliefs at the end of period* 1 *are*

$$\mu^e_{L1}(i_{k1}) = \begin{cases} \dfrac{\frac{1}{2} + \hat{\mu}_L}{2} & \text{if } i_{k1} = \frac{1}{2} \\ \dfrac{i_{k1} + \hat{\mu}_L}{2} & \text{if } i_{k1} < \frac{1}{2} \end{cases}. \tag{10}$$

The proof and all details of the equilibrium choices of Proposition 6 can be found in the Appendix.

An immediate consequence of Proposition 6 is

COROLLARY 3:

$$\hat{a}^{SR} = \hat{a} = \frac{1}{2}\sqrt{\frac{A}{2\lambda}\left(\frac{1}{2} - \hat{\mu}_L\right)}. \tag{11}$$

Corollary 3 follows from the proof of Proposition 6, in which $\hat{a}^{SR}$ was calculated, but it can also be derived from the following reasoning. At the lowest possible ability level that still yields reelection, the median voter has to be indifferent between reelection and deselection. Hence, as the officeholder is strictly less attractive for voters with $i > 1/2$, the officeholder with ability $\hat{a}^{SR}$ will just receive 50 percent of the votes at the end of period 1. With $s_1 = 50$ percent, the SR-rule does not impact policy choices and reelection chances when compared to standard elections. Hence, for the ability level $\hat{a}^{SR}$, strategies and beliefs are equal to the ones in standard elections in all periods (including the last election), and the officeholder is just reelected with the minimal possible vote share. As the officeholder is just reelected under standard elections at ability levels $\hat{a}$, $\hat{a}^{SR} = \hat{a}$ must hold.

IV. Comparison and Competition with Score-Replication Rule

We next compare standard elections and elections with the SR-rule. We obtain the following result.

THEOREM 1: *Elections with the SR-rule yield higher expected welfare than standard elections.*

This theorem is a consequence of the preceding propositions, including the proof of Proposition 6 and Corollary 3. The SR-rule does not affect the expected ability of politicians in office, but it does reduce the possibilities of reelected incumbents to indulge in their own preferences.

More specifically, the SR-rule reduces the ideological bias of officeholders, since otherwise, even officeholders with above-average ability would risk deselection. The reason is that larger deviations from the median voter's ideological position signal that the most preferred ideological position of the officeholder is more extreme. Thus, the SR-rule reduces policy polarization. In turn, the SR-rule does not affect the average ability officeholders because officeholders in the second term adopt more moderate policies and thus, officeholders with ability above $\hat{a}^{SR} = \hat{a}$ are reelected.

So far, the SR-rule is set by the public. Yet, one could also allow candidates for office to *offer* the score-replication rule.[17] If candidates can choose between offering the SR-rule or not before the election starts, the following result is an immediate consequence.

COROLLARY 4: *Suppose candidates can either offer the SR-rule or not. Both candidates L and R, then, commit to the SR-rule in the first election.*

Corollary 4 is a simple consequence of the fact that offering the SR-rule yields higher expected utility for the median voter. As a consequence, standard Bertrand-type reasoning yields that often, the SR-rule is the only equilibrium outcome. Not offering the SR-rule, if the other candidate offers it, would entail no chance to be elected. Conversely, if the other candidate does *not* offer the SR-rule, the first candidate can ensure election by offering it himself. Hence, in the basic version of our model, it does not matter whether the public or candidates offer the SR-rule during campaigns.[18]

[17] For such offers to be legally binding, one has to turn them into so-called "political contracts." See Gersbach (2012) for the structure and function of political contracts.

[18] Once could allow a richer set of offers, with different types of score-replication rules with which candidates could compete. In settings in which parameters of elections are changing (e.g., number of and information about candidates and type of policies), such competition with score-replication rules might be particularly advantageous, as current parameters of elections will be incorporated by candidates. These issues are left for future research.

V. On Strategic Choices

A. *Strategic Choices of Candidates*

We assumed that the value of office is sufficiently large for candidates. This implies that candidates pursue ideological policies in the second term but not in the first one. The reason is as follows. In equilibrium, if an officeholder is reelected for the second term, he has above-average ability. Thus, he can ensure reelection to the third term by a judiciously chosen ideological position in the second term.

If now an officeholder deviated from the centrist policy in the first term, to move toward his preferred position, in order to lower the vote share and thus his future reelection hurdle, then he would strictly lower his reelection chances for a second term, since his ability is not known yet. With more ideological policy choices and the associated beliefs that he is more ideological than the average, he would risk to be deselected when he turns out to be of intermediate ability, close to the threshold in equilibrium.[19] If policy makers sufficiently value office, the marginal benefits from deviating toward the preferred ideological position are smaller than the expected marginal losses of the value of office-holding over two terms. This is established in the Proof of Proposition 6 (Step 9).[20]

B. *Strategic Voting*

So far, we assumed that citizens vote sincerely. We next investigate whether and how the outcomes may change if we consider strategic voting. We can immediately observe that citizens in the second and third period cannot gain from strategic voting, since it would only help to elect a candidate who will generate lower expected utility.

However, there are strong incentives for strategic voting in the first period, which we now examine. Suppose that a left-wing candidate who has chosen $i_{k1} = 1/2$ in the first period turns out to be of ability $a > \hat{a}^{SR}$. With sincere voting, he would attract all voters on the left of some voter s_1 with $s_1 > 1/2$. We also denote the expected policy choice in the second term, depending on the vote share s_1, by $E\left[i_{k2}(s_1)\right]$, which will be below $1/2$ and is strictly increasing in s_1 due to the SR-rule. The different cases for calculating $E\left[i_{k2}(s_1)\right]$ are given by Proposition 10 in the Appendix. Since a higher vote share translates in less ideological left-wing positions in the second term, there are two motives for voting strategically:

- Voters with $i > E\left[i_{k2}(s_1)\right]$ may want to vote for the left-wing candidate k in order to decrease the left-wing ideological bias in the second term—i.e., to increase $E\left[i_{k2}(s_1)\right]$.

[19] If ability were already known when initial policies are chosen, candidates would pursue ideological policies in the first period to reduce their vote share, but they would not go beyond their preferred ideological position.

[20] If the value of officeholding is low, candidates might already pursue ideological policies in the first period. Let us take the extreme example in which officeholders are solely interested in ideological policies and public projects. Since a deselection would allow the opposing candidate to pursue ideological policies, officeholders are still interested in reelections to avoid such cases. However, the gain from ideological policies in the first period has a large weight, and thus deviations from the median position can occur. The magnitude of ideological policy choices in the first period crucially depends on the distribution of abilities and on the model parameters.

- Voters with $i < E\left[i_{k2}(s_1)\right]$ may want to support the right-wing challenger in order to lower s_1 and to reduce $E\left[i_{k2}(s_1)\right]$.

The question now is which equilibria emerge under strategic voting. Suppose that the left-wing candidate k wins the election and he has ability $a > \hat{a}^{SR}$.[21] Suppose also that his vote share is $\tilde{s}_1 > 1/2$. The expected policy position in the second period is given by $E\left[i_{k2}(\tilde{s}_1)\right] < 1/2$. Since there are no incentives for strategic voting in the second period, Proposition 10 provides the formulas for the different cases. According to the above observations, such a constellation can only be an eqilibrium under strategic voting if the following holds:

- All voters with $i > E\left[i_{k2}(\tilde{s}_1)\right]$ support candidate k since they want to increase the vote share to move $E\left[i_{k2}(\tilde{s}_1)\right]$ to the right.
- Similarly, all votes with $i < E\left[i_{k2}(\tilde{s}_1)\right]$ support the opposing candidate k' in order to decrease the vote share of candidate k and to move $E\left[i_{k2}(\tilde{s}_1)\right]$ to the left.

Hence, the constellation can only be an equilibrium if the vote share $\tilde{s}_1$ is equal to $1 - E\left[i_{k2}(\tilde{s}_1)\right]$ since, under strategic voting, precisely voters in the interval $\left[E\left[i_{k2}(\tilde{s}_1)\right], 1\right]$ vote for candidate k. The next proposition shows that there is indeed a unique value $\tilde{s}_1$ that fulfills this condition, which in turn characterizes the equilibrium under strategic voting.

PROPOSITION 7: *Suppose $a > \hat{a}^{SR}$ and that politician k is the incumbent. With strategic voting, the equilibrium value of the vote share in the first period, denoted by $\tilde{s}_1$, is the unique solution of*

(12) $$E\left[1 - i_{k2}(\tilde{s}_1)\right] = \tilde{s}_1.$$

Moreover,

(*i*) *voters with $i > \tilde{s}_1$ support k;*

(*ii*) *voters with $i < \tilde{s}_1$ support k'.*

Proposition 7 follows from the observation that equation (12) has a unique solution, since $E\left[1 - i_{k2}(\tilde{s}_1)\right]$ is strictly decreasing in $1 - \tilde{s}_1$. The explicit solution can be determined by the same procedure as in Proposition 6 and yields the value $\tilde{s}_1 < 1/2$. Hence, compared to sincere voting, moderate voters in $\left[\tilde{s}_1, s_1\right]$ still support the left-wing candidate who has high ability. However, more extreme voters now vote strategically to influence the vote share of the winning candidate. Right-wing voters with $i > s_1$ want to increase the vote share, while left-wing

[21] There can be no equilibria under sincere or strategic voting in which the challenger wins, since his expected ability is zero.

voters with $i < \tilde{s}_1$ want to lower it. Whether strategic voting leads to higher or lower vote shares—and thus to more or less moderate ideological positions in the future—depends on the parameters and, in particular, on the weight λ.[22]

VI. Extensions

A. *Welfare-Reducing Incumbency Advantages*

In the first extension, we consider the basic scenario with welfare-reducing incumbency advantages. These are advantages that solely stem from the fact that a politician is in office. They are generally detrimental to society and come from a variety of sources. Often, they are connected to better opportunities for an officeholder to attract votes compared to a challenger, even if this challenger has similar personal characteristics, such as competence. A simple approach to this issue is to assume that there is a share θ $(0 < \theta < 1/2)$ of impressionable voters whose support can be attracted by investing a part of the output of a public project. Specifically, let γ $(\gamma > 0)$ be the amount of the public project that is needed to attract impressionable voters.[23] The level of public project provision is reduced to $g_t = a_k - \gamma$ in such cases if an incumbent with ability a_k is in power. Suppose also that the decision on investing to attract impressionable voters has to be taken at the beginning of a term. We obtain the following result.

PROPOSITION 8: *For standard elections*,

(i) *In the first period*, *all incumbents will invest* γ *to attract impressionable voters, and will choose* $i_{k1} = 1/2$.

(ii) *New officeholders in the second period will invest* γ *to attract impressionable voters, and select* $i_{k2} = 1/2$.

(iii) *In the second period, reelected incumbents will invest* γ, *unless they can secure reelection without investing. They will indulge in their ideological preferences if their ability is above a particular threshold.*

(iv) *In the last period*, *incumbents will not invest*.

The proof of Proposition 8 is straightforward.[24] Suppose now that the SR-rule is applied instead of standard election rules. We observe that the general pattern of investments to attract impressionable voters is the same as in Proposition 8. Again, however, the SR-rule can limit the deviations of second-term officeholders from moderate ideological policy choices.

[22] Examples are available upon request.

[23] For the sake of simplicity, we assume that this is a (non-divisible) zero/one decision (investment or no-investment) to attract impressionable voters.

[24] However, the policy choices are involved and the precise choice of reelected incumbents in the second period depends on the parameters.

B. *Initial Advantages*

We have assumed that incumbents who have no chance to be reelected will not stand for reelection. Thus, new candidates have a 50 percent chance of being elected in such circumstances. Suppose now, however, that all incumbents stand for reelection independently of their reelection prospects. First-time officeholders may then obtain a large share of votes if their opponent's ability is low. In such situations, the SR-rule might entail a negative effect on welfare, as reelection becomes much harder and quite able officeholders might be deselected. Such disadvantages can be avoided by the RSR-rule, introduced in Section IIIA. With this rule, the initial election result has no impact on future policies and elections. Hence we obtain the following result.

PROPOSITION 9: *The RSR-rule improves welfare compared to standard elections, no matter whether low-ability officeholders stand for reelection.*

C. *Noise and the Selection of Challengers*

The equilibria derived so far have the deterministic property that in the second term, an incumbent can secure reelection by judiciously choosing his policy, since he can precisely calculate the vote share he will obtain. A challenger has zero chance of being elected. For the sake of completion, one should note that more patterns emerge if there is additional noise about the ability of politicians and if there are entry barriers.

There are several ways to introduce noise so that we can limit ourselves to a simple framework (see, e.g., Morris and Shin 2001) and only provide two short examples.[25] We assume that ability a is randomly drawn from the real line, with realization being normally distributed, with mean 0 and variance σ_a^2, denoted by $N(0, \sigma_a^2)$. A politician k observes a private signal $y_k = a_k + \varepsilon_k$, with ε_k being independently normally distributed, with mean 0 and variance σ_s^2. Hence, when a politician observes the signal y_k, he considers ability to be distributed normally, with the derived mean $E[a_k | y_k]$ and variance σ_p^2.

We continue to assume that officeholders learn their true ability when they are in their first term. Suppose now that an incumbent k has been reelected in the first term, with some ability a_k and vote share s_1,[26] and that he faces a challenger. The challenger k' also observes a private signal $y_{k'} = a_{k'} + \varepsilon_{k'}$, with $\varepsilon_{k'}$ again being independently normally distributed, with mean 0 and variance σ_s^2, and he forms the a posteriori distribution $N(E[a_k | y_{k'}], \sigma_p^2)$, if he observes $y_{k'}$.

As long as the public does not observe the signal and as long as there are no entry costs, nothing changes in our equilibrium considerations, since the citizens' expected ability of a new candidate is still zero. The situation would change substantially if

[25] A comprehensive analysis is outside the scope of this article. Details for the examples are available upon request.

[26] The vote share s_1 has to be determined in the overall equilibrium of the game. Here, we address the impact of noise on the crucial second period.

the public also obtained a signal about the challenger's ability. Moreover, competition would be heightened if challengers faced entry costs and thus would only enter if the chances to enter office times the benefits from holding office exceed the entry costs.

We illustrate each extension with an example. First, if the public can also observe the signal about the challenger, the officeholder faces a risky choice in the second term. The precise choices depend on the precision of the ability signal and the expected benefits from holding office. As an example, we report one particularly interesting case.

EXAMPLE 1: *Suppose that the officeholder's ability has turned out to be high and let $\sigma_s = 0$ (i.e., let the signal be completely informative). Then, for suitable values of the model parameters, the officeholder will choose*

- $i_{k2} = \mu_L$ *if the simple majority rule is applied*;
- $i_{k2} = 1/2$ *if the SR-rule is applied.*

The intuition for the result runs as follows. With a signal, the most important change is that the incumbent does not know the signal when making his policy decision. The fact that there is a positive probability that the signal will communicate a strictly higher expected ability than that of the incumbent implies that the incumbent's chance to be reelected is strictly smaller than one. If the incumbent partly or fully reveals his ideological bias, this can only reduce his attractiveness for voters. If attractiveness is strictly reduced, then his chances to be reelected will be smaller. Hence it is better for him not to reveal his best point, but to pool at $1/2$. For small b, however, a small gain in reelection chances from pooling at $1/2$ may not offset the advantages of playing the ideal policy in term 2. This happens for sufficiently able officeholders. For normally distributed abilities, there always exist such officeholders. In this case—when deviating from pooling at $1/2$ is optimal if the simple majority rule is applied—the SR-rule does have a strong effect; no matter how small the decrease in attractiveness, the resulting vote share for reelection will be strictly lower than the vote share at the end of the first round, when the ideal point is still unrevealed. Hence, certain deselection follows when a deviation from $i_{k2} = 1/2$ strictly reduces the vote share s_2 (i.e., k is forced to choose $i_{k2} = 1/2$). Hence, there are circumstances in which score-replication rules eliminate any ideological biases in the second term.

Second, we consider a situation in which there are entry costs for entrants in the second period. Let $\delta > 0$ be entry costs and let us assume there is a pool of potential challengers $k_j', j \in \{1, \ldots, n\}$ with $n \geq 1$. Each of these potential challengers receives a signal about these abilities

$$y_{k_j'} = a_{k_j'} + \varepsilon_{k_j'}$$

which is private information, but becomes public once candidates enter the campaign against the incumbent. The signals have the same structure as before. Potential challengers decide whether to compete in the right-wing party to become the challenger.

For the sake of simplicity, we assume that each of the entering candidates has the same chance of being selected by his party. In such circumstances, we observe two additional effects. First, only potential challengers who receive sufficiently positive signals about ability are willing to enter. As a consequence, officeholders will be cautious regarding their choice of ideological policies in order to preserve their chances to be reelected. In turn, these cautious policy choices will further tighten the threshold of the signal above which the potential challenger will enter. This effect improves the efficiency of both the simple majority rule and the SR-rule.

Moreover, with entry barriers, high-ability candidates are drawn into the race, which makes it harder to obtain vote shares above 50 percent. Hence, weaker forms of score-replication rules may suffice to eliminate ideological biases in the second term. Since the examination of the entry game is extremely complex, a thorough investigation is left to future research. One can construct examples in which weaker forms of SR-rules may suffice to eliminate all biases. For that purpose, we define the $1/2$-score-replication rule as the rule under which an officeholder needs to replicate a vote share of 50 percent plus $1/2$ of the vote share above 50 percent received in the previous election.

EXAMPLE 2: *Suppose the officeholder's ability has turned out to be high and let* $\sigma_s = 0$ *and consider one potential entrant* $n = 1$. *Then, for suitable values of* δ *and* b, *the officeholder will choose*

- $i_{k2} = \mu_L$ *if the simple majority rule is applied*;
- $i_{k2} = 1/2$ *if the* $1/2$*-score-replication rule is applied.*

D. *Score-Replication Rules in Other Electoral Systems*

Score-replication rules could also be applied to other electoral environments and to proportional election schemes that are prevalent in continental Europe, in particular. For instance, suppose that seats in a legislature are distributed to parties according to the vote share they receive for their list of candidates. The voters can modify any party's list of candidates, and thus some candidates are more popular than others within the same party list. This type of "open list" is common in most European democracies. Given the number of seats assigned to a party, the candidates with the most individual votes take office. Typically, incumbents will receive a considerably higher number of individual votes than new candidates on the list. Under score-replication, an incumbent who matches the votes received in the last election is reelected, provided his party obtains the necessary amount of seats.

Hence, with a score-replication rule, ceteris paribus, there will be higher turnover of officeholders, and thus the average number of terms in office will be lower. Newcomers will have better chances to enter parliament. The overall vote share of the parties, however, may be little affected, as score-replication rules only affect competition for seats *within* the party. In anticipation of these effects, parliament party factions will choose policies that particularly boost their score in the next election.

As a result, one would expect that more moderate positions emerge. However, if more than two parties compete for seats, this moderation is less clear. A thorough analysis of this issue is left to future research.

E. *Exogenous Score-Replication Rules*

The score-replication rules in this paper are endogenous (i.e., they are determined by the vote share an incumbent receives at the end of his first term). An alternative rule would be to use exogenously determined score-replication rules, which would rule out strategic voting behavior. Such a predetermined score-replication rule could be obtained by taking a suitable average over all endogenously determined equilibrium vote shares. The predetermined vote share needed to stay in office would increase with the number of terms in office. An exogenously determined score-replication rule would entail two inefficiencies. First, it would cause the deselection of officeholders with moderately positive abilities above the threshold we obtain under endogenous score-replication rules. Second, it would allow officeholders with high ability to indulge more in their ideological preferences in the second term. Still, an optimally selected fixed score-replication rule would yield higher welfare than standard elections.[27] Moreover, exogenously determined score-replication rules are much less complex and thus easier to understand and apply than endogenous rules.

VII. Discussion and Conclusion

We have provided a first analysis of the score-replication rule. Our results show that experimenting with score-replication rules would enrich the opportunities of democracy to achieve socially desirable outcomes for its citizens. Numerous extensions can be pursued. For instance, one might consider a longer time horizon or even an infinite horizon to study variants of the score-replication rule. Moreover, one could add exogenous shocks to reelection chances and see how they affect the comparison between standard elections and elections under the score-replication rule. These issues are left for future research.

Of course, score-replication rules would require changes of the laws and constitutional clauses that govern democratic electoral processes. Moreover, numerous issues deserve further scrutiny.

They can be classified in three groups. First, while score-replication rules work well in our model, there is still some concern that a slight dent in the popularity of an officeholder may be punished too harshly if it occurred for reasons that are beyond the control of this officeholder. This might be the case if the economy is affected by negative macroeconomic shocks. In such cases, an officeholder with above-average ability may be deselected. In extreme cases, an officeholder who obtains 60 percent in one election could be deselected with 59 percent of the votes in the next election,

[27] This follows from the observation that a fixed score-replication rule $(1/2) + \epsilon$ for some small $\epsilon > 0$ marginally improves welfare. The reason is that it constrains the ideological choice of officeholders who are reelected as long as their ability is not too high. The welfare losses only concern an arbitrarily small subset of officeholders who have intermediate ability but are not reelected. Gains are of order ϵ while costs are of order ϵ^2.

for instance. To lessen such concerns, one could implement weaker variants of the score-replication rule. There is a wide range of such weaker forms. For instance, the vote-threshold needed for reelection could be reduced by some margin compared to the highest score in the past. Or one could set the threshold equal to the average score in all previous elections. Moreover, as we have discussed, score-replication rules might be applied only after the first reelection to reduce the pressure to match a high score too early.

Second, the requirement to match previous election scores could encourage officeholders to focus on policies that deliver short-term results. This is a concern for any election procedure. It is not clear whether score-replication rules would strengthen or reduce such short-termism. On the one hand, an initial high election score could strengthen short-termism, as officeholders need to replicate high election scores in the next reelection bid. On the other hand, officeholders might attempt to have high reelection scores only towards the end of their planned time in office, which might leave room for farsighted policy making at the beginning of the term. To lessen such concerns, it could be advisable to use score-replication rules that become effective only after the first reelection or to use other weaker forms of score-replication rules.

Third, score-replication rules would affect a variety of strategic interactions between candidates, parties, voters, and interest groups in election races. For instance, interest groups opposing the incumbent may encourage a third candidate to run, to make it more difficult for the officeholder to attain his previous score. If the winner is determined by a run-off between the best two candidates (as, for example, in the presidential election in France after a first election round), then the problem can easily be solved by applying the score-replication rule only to the run-off round. If there is only one election round and the winner is determined by the highest vote share, the score-replication rule can be applied as follows. Suppose an incumbent k has to fulfill the score $s_1 > 1/2$ from the previous election in which only two candidates competed. Suppose that incumbent k now faces the challenger k' and a third candidate k'' who only enters to reduce the vote share of candidate k, and thereby enforce deselection. Suppose that they receive vote shares s_{2k}, $s_{2k'}$, and $s_{2k''}$ with $s_{2k} > s_{2k'} > s_{2k''}$. Then let us define $\hat{s}_{2k} = s_{2k}/(s_{2k} + s_{2k'}) = s_{2k}/(1 - s_{2k''})$. The score-replication rule could then be modified to account for strategic challenger entry in the following way: The incumbent k is reelected if $\hat{s}_{2k} > s_1$ (i.e., if he receives a larger vote share than s_1 among the two best candidates). This would alleviate strategic entry of challenger.

While many further issues should be considered, history-bound reelections may yield new forms of democracy that go beyond the simple model studied in this paper. In particular, the score-replication rule could be applied to many areas—not only to two-party systems—or to plurality majority voting, as discussed in Section VID.

History-bound reelections treat incumbents and challengers differently, which is justified by the fact that they typically differ regarding their chances to be elected, even if they have the same preferences and abilities. There is no denying, however, that history-bound reelections are a shift away from the basic principle that in democracy, the vote-threshold for entering a public office should be the same for all candidates. However, other rules already deviate from this principle, such as term limits, for example.

Overall, it is clear that such new types of political competition require a wide range of further considerations and analyses.

Appendix

List of Notation

Symbol	Meaning
$t = 1,2,3$	Period
τ	Term of an officeholder
$i \in [0,1]$	A voter and this voter's preferred policy
k, k'	Officeholders or candidates for office
I	A chosen ideological policy in some given period
i_{kt}	Policy choice of officeholder k in period t
P	Public project undertaken in some period
g_t	Level of provision of a public project in period t
a_k	Ability of officeholder k
$A > 0$	Ability of an officeholder is drawn from a random variable distributed uniformly in $[-A, A]$.
v_{it}	Utility function of voter i in period t
$\lambda > 0$	Weight voter i puts on utility from public project provision versus utility from ideological policies
V_i	Expected lifetime utility of voter i
μ_L, μ_R	Ideal ideological position of a left-wing, resp. right-wing, politician
μ_k	Ideal position of a politician k
$\hat{\mu}_L, \hat{\mu}_R$	Most extreme possible left-wing, resp. right-wing, ideal position
$b > 0$	Benefits derived by an officeholder from being in power for one period
v_k	Utility of an officeholder k in a particular period
V_k	Expected lifetime utility of politician k
$\mu_{L2}^e(i_{k2})$	Expected ideal position of a left-wing officeholder in the second period, once the electorate views the policy choice i_{k2}
s_τ	Vote share required for reelection for term $\tau + 1$ under the SR-rule
s_1^*	Equilibrium vote share in period 1
$\hat{a}$	Minimum ability that a first-period officeholder must have in order to secure reelection under standard election rules
$\hat{a}^{SR}$	Minimum ability that a first-period officeholder must have in order to secure reelection under the SR-rule
γ, θ	Investment (γ) to attract θ impressionable voters

PROOF OF PROPOSITION 1:

We verify the claim for two different constellations.

Step 1: Suppose that $\mu_{L2}^e(i_{k2})$ is flat on some interval $[\underline{i}, \overline{i}]$ with $\hat{\mu}_L \leq \underline{i} < \overline{i} \leq 1/2$ and strictly increasing in the rest of the interval $[\hat{\mu}_L, 1/2]$ for equilibrium policy choices. Then, any policy choice i_{k2} in $[\underline{i}, \overline{i}]$ yields the same reelection chances, since reelection chances can only depend on ability and the beliefs $\mu^e_{L2}(i_{k2})$. Hence, the best responses of the left-wing incumbent can be characterized as

$$(13)\qquad i_{k2}\begin{cases}\geq \bar{i} & \text{if } \mu_k \in \left(\bar{i},\frac{1}{2}\right] \\ = \mu_k \text{ or } \geq \bar{i} & \text{if } \mu_k \in \left[\underline{i},\bar{i}\right] \\ \leq \underline{i} \text{ or } \geq \bar{i} & \text{if } \mu_k \in [\hat{\mu}_L,\underline{i})\end{cases}.$$

If the best response of an officeholder with $\mu_k \in \left[\underline{i},\bar{i}\right]$ lies within $\left[\underline{i},\bar{i}\right]$, he will choose $i_{k2} = \mu_k$. This choice does maximize his current utility, and all choices involve the same reelection chances. This leads to the contradiction due to the assumed flatness.

Step 2: Suppose now $\mu^e_{L2}(i_{k2})$ is strictly decreasing in $\left[\underline{i},\bar{i}\right]$. Then, again, the policy maker with $\mu_k \notin \left[\underline{i},\bar{i}\right]$ would choose policies outside the interval $\left(\underline{i},\bar{i}\right)$. The officeholder with $\mu_k \in \left[\underline{i},\bar{i}\right]$ would either choose policies outside of the interval $\left(\underline{i},\bar{i}\right)$ or within $\left(\underline{i},\bar{i}\right)$. If two officeholders k and k' with $\mu_k < \mu_{k'}$ chose a best response i_{k2} and $i_{k'2}$ in $\left(\underline{i},\bar{i}\right)$, then $i_{k2} \leq i_{k'2}$. This follows from the observation that $i_{k2} > i_{k'2}$ would lead to a contradiction. In such a situation, the officeholder could increase his expected utility by choosing $i_{k'2}$.

To sum up, if policies in $\left(\underline{i},\bar{i}\right)$ are selected in equilibrium, the belief function μ^e_{L2} cannot decrease.

Step 3: The preceding considerations can be applied to any constellation of the belief function $\mu^e_{L2}(i_{k2})$ in which some part is not strictly increasing on the equilibrium path. ∎

LEMMA 2: *Consider two possible left-wing officeholders k and k' in their second term, with revealed abilities $a_k = a_{k'}$. Suppose no information about the ideal policies has been revealed and that $\mu_k < \mu_{k'} < 1/2$. In any equilibrium,*

$$(14)\qquad \mu_k \leq i_{k2} \leq i_{k'2}.$$

PROOF OF LEMMA 2:

Lemma 2 follows by contradiction. We first observe that no officeholder k chooses a policy $i_{k2} < \mu_k$. The officeholder could increase his own utility in such circumstances by choosing μ_k and, by doing so, the voters' belief about his ideological position would not become less attractive to the median voter as is implied by Proposition 1 and Refinement 1. Hence, suppose now $\mu_k \leq i_{k'2} < i_{k2}$. First, if $\mu^e_{L2}(i_{k'2}) = \mu^e_{L2}(i_{k2})$, the reelection chances of both types of officeholders k and k' are the same. But then officeholder k could increase his own utility by choosing $i_{k'2}$ instead of i_{k2}, because he would reduce his own utility loss from choosing a policy that is different from his ideal point without affecting his reelection chances. Hence, we obtain a contradiction. Second, let $\mu^e_{L2}(i_{k'2}) < \mu^e_{L2}(i_{k2})$. If the reelection chances of k are higher than those of k', the policy choice of k' cannot be optimal, because officeholders maximize their reelection chances. ∎

PROOF OF PROPOSITION 2:

We show that the strategies and beliefs specified in the proposition do indeed constitute an equilibrium. We derive the optimal choice of an officeholder with ability a_k and ideal point μ_L. Given some belief function $\mu^e_{L2}(i_{k2})$, the expected utility of the median voter for the third period from reelecting the incumbent is

$$\lambda a_k - \left|\mu^e_{L2}(i_{k2}) - \frac{1}{2}\right|.$$

The expected utility of the median voter in the third period from selecting a right-wing challenger is

$$\lambda E[a_{k'}] - E\left[\left|\frac{1}{2} - \mu_R\right|\right].$$

The officeholder is reelected if the median voter is at least indifferent between the incumbent and the challenger—that is, if

$$\lambda a_k - \frac{1}{2} + \mu^e_{L2}(i_{k2}) \geq \frac{1}{4} - \frac{\hat{\mu}_R}{2}. \tag{15}$$

Moreover, we know from the proof of Lemma 2 that

$$\mu_k \leq i_{k2}. \tag{16}$$

With the officeholder's policy choices being as stated in the proposition, voters would rationally form the belief

$$\mu^e_{L2}(i_{k2}) = \begin{cases} i_{k2} & \text{if } \frac{1}{2} - 2\lambda a_k < i_{k2} \leq \frac{1}{2} \\ \frac{1 - 4\lambda a_k + 2\hat{\mu}_L}{4} & \text{if } i_{k2} = \frac{1}{2} - 2\lambda a_k \end{cases}. \tag{17}$$

Hence, for $i_{k2} \in ((1/2) - 2\lambda a_k, 1/2]$, inequality (15) is strict. For $i_{k2} = (1/2) - 2\lambda a_k$, we obtain

$$\lambda a_k - \frac{1}{2} + \frac{1}{4} - \lambda a_k + \frac{1}{2}\hat{\mu}_L = \frac{1}{2}\hat{\mu}_L - \frac{1}{4} = \frac{1}{4} - \frac{\hat{\mu}_R}{2}.$$

Hence, (15) is fulfilled as an equality. Thus, all officeholders are reelected. There are no profitable deviations by officeholders. Officeholders with $\mu_L \in ((1/2) - 2\lambda a_k, 1/2]$ can implement their most preferred policy and can still secure reelection. Officeholders with $\mu_L \in [\hat{\mu}_L, (1/2) - 2\lambda a_k]$ would be deselected if they chose a policy $i_{k2} < (1/2) - 2\lambda a_k$.

Finally, we note that out-of-equilibrium beliefs are constructed under the assumption that any accidental choice $i_{k2} < (1/2) - 2\lambda a_k$ would stem from officeholders with $\mu_L \leq i_{k2}$, and that such choices are equally likely among those candidates. This satisfies Refinement 1. This completes the proof of the existence of the equilibrium. Uniqueness follows from the fact that beliefs μ^e_{L2} are non-decreasing on the whole interval $\left[\hat{\mu}_L, 1/2\right]$, due to Proposition 1 and Refinement 1. ∎

PROOF OF PROPOSITION 3:

We first show that the described equilibrium exists. Suppose that the ability of the officeholder turns out to be a_k. By selecting $i_{k2} = 1/2$, the expected utility of the median voter, V_m, for the third period is

$$V_m = \lambda a_k - \left| \frac{1}{2} - \frac{\frac{1}{2} + \hat{\mu}_L}{2} \right|. \tag{18}$$

Hence, the officeholder is reelected if and only if $a_k \geq 0$, because the expected utility from selecting a challenger is

$$-\left| \frac{1}{2} - \frac{\frac{1}{2} + \hat{\mu}_R}{2} \right| = -\left| \frac{1}{2} - \frac{\frac{1}{2} + \hat{\mu}_L}{2} \right|.$$

Hence, from the perspective of the beginning of the second period, the probability of being reelected is $1/2$. If the officeholder chooses some position $i_{k2} < 1/2$, he will be reelected if and only if

$$\lambda a_k - \left| \frac{1}{2} - \frac{i_{k2} + \hat{\mu}_L}{2} \right| \geq -\left| \frac{1}{2} - \frac{\frac{1}{2} + \hat{\mu}_R}{2} \right|, \tag{19}$$

and thus if and only if

$$a_k \geq a^{crit}(i_{k2}) := \frac{1}{4\lambda}(1 - 2\,i_{k2}). \tag{20}$$

Because $a^{crit}(i_{k2}) > 0$ for all $i_{k2} < 1/2$, the expected reelection probability of the officeholder, given by

$$\frac{A - a^{crit}(i_{k2})}{2A}, \tag{21}$$

is strictly below $1/2$. Hence, the deviation is not profitable, because reelection concerns determine the officeholder's behavior.

We next address the uniqueness of the equilibria. We first note that out-of-equilibrium beliefs are not unique, and the uniqueness only concerns the equilibrium strategies and beliefs. Suppose that there exists an equilibrium in which a non-empty set of left-wing officeholders chooses the ideological policy $i_{k2} < 1/2$. Because beliefs $\mu^e_{L2}(i_{k2})$ are non-decreasing (Refinement 1), the choice i_{k2} cannot be optimal for officeholders with $i_{k2} < \mu_L \leq 1/2$. Hence, these officeholders will choose some other policies in the candidate equilibrium. Suppose they choose $i'_{k2} > i_{k2}$.[28] Because the reelection probability strictly increases in $\mu^e_{L2}(i_{k2})$ and all officeholders maximize their reelection chances, we must have

$$\mu^e_{L2}(i_{k2}) = \mu^e_{L2}(i'_{k2}).$$

Because beliefs are a non-decreasing function of ideological policies, we must have that the beliefs are constant in $[i_{k2}, i'_{k2}]$. However, given such beliefs, only officeholders with $\mu_L < (i_{k2} + i'_{k2})/2$ would choose i_{k2}, while only officeholders with $\mu_L > (i_{k2} + i'_{k2})/2$ would choose i'_{k2}, because within $[i_{k2}, i'_{k2}]$, their reelection chances are the same and officeholders choose policies closer to their ideal point. This is a contradiction to $\mu^e_{L2}(i_{k2}) = \mu^e_{L2}(i'_{k2})$. Hence, the equilibrium is unique. This completes the proof. ∎

PROOF OF PROPOSITION 4:

We establish the existence of the equilibrium. Because both candidates will generate the same expected utility for the median voter, they both have a chance of $1/2$ to be elected for the first period. Without loss of generality, suppose that the left-wing candidate has been elected. If he chooses $i_{k1} = 1/2$ independently of his ideal point μ_L, then

$$\mu^e_{L1}\left(\frac{1}{2}\right) = \frac{\frac{1}{2} + \hat{\mu}_L}{2}.$$

Suppose now that the officeholder turns out to have ability a_k. If $a_k < 0$, the officeholder will be deselected after period 1.[29,30] If $a_k \geq 0$, the median voter's expected utility from reelecting the officeholder for the subsequent periods is

$$V_m = 2\lambda a_k - \left|\left(\frac{1}{2} - 2\lambda a_k\right)\frac{\frac{1}{2} - 2\lambda a_k - \hat{\mu}_L}{\frac{1}{2} - \hat{\mu}_L} + \frac{\frac{1}{2} - 2\lambda a_k + \frac{1}{2}}{2}\frac{\frac{1}{2} - \frac{1}{2} + 2\lambda a_k}{\frac{1}{2} - \hat{\mu}_L} - \frac{1}{2}\right|$$
$$- \left|\frac{1}{2} - \frac{\frac{1}{2} + \hat{\mu}_L}{2}\right|.$$

Note that the median voter anticipates that an officeholder with $a_k \geq 0$ will be reelected for the third period. Deselecting the current officeholder and electing a right-wing challenger yields

[28] The logic can easily be extended to other policy choices.

[29] If officeholder k with $a_k < 0$ were reelected for the second period, he would not be reelected for the third period (according to Proposition 1). The expected utility from following this strategy is strictly lower than the utility from deselecting the officeholder. The latter produces the expected utility derived in equations (22) to (24) below.

[30] This follows from Proposition 3. A newly elected officeholder will choose $i_{k2} = 1/2$, and thus will generate higher expected utility than the current officeholder, regardless of the latter's choice of the ideological position in the second period.

(22) $$V'_m = -\left|\frac{\frac{1}{2} + \hat{\mu}_R}{2} - \frac{1}{2}\right| + \frac{1}{2}\lambda E\left[a'_k \,|\, a'_k > 0\right]$$

(23) $$= -\left|\frac{\frac{1}{2} + \hat{\mu}_R}{2} - \frac{1}{2}\right| + \lambda\frac{A}{4}$$

(24) $$= -\frac{1}{2}\left(\frac{1}{2} - \hat{\mu}_L\right) + \lambda\frac{A}{4},$$

where we have used the fact that a newly elected officeholder will choose $i_{k2} = 1/2$ in the second period and will be reelected if and only if $a'_k \geq 0$. For the last period, any officeholder will choose his ideal point. Comparing V_m and V'_m yields that the officeholder is reelected if and only if

(25) $$a_k \geq \hat{a}_L := \frac{1}{2}\sqrt{\frac{A}{2\lambda}\left(\frac{1}{2} - \hat{\mu}_L\right)}.$$

Given the beliefs, an officeholder who deviates by selecting $i_{k1} < 1/2$ would strictly reduce V_m and thus raise the critical ability threshold above which he would be reelected. Hence, for the given beliefs, the choice $i_{k1} = 1/2$ is optimal for officeholders independently of their ideal point. This establishes the existence of the equilibrium.

Regarding uniqueness, we again note that out-of-equilibrium beliefs are not unique, but with the refinement, we claim that equilibrium strategies and equilibrium beliefs are unique. This claim can be proven with the same arguments as the ones used in Proposition 3. ∎

PROOF OF COROLLARY 2:

In the third period, the median voter's expected utility from ideological policies is the same for both officeholders. Officeholder k, however, produces higher utility from the public project provision λa_k compared to $\lambda a_{k'}$.

We thus turn to the second period. Suppose first that $2\lambda a_{k'} \geq (1/2) - \hat{\mu}_L$. Because officeholders will choose their preferred ideological position, higher ability directly translates into higher utility of the median voter.

Suppose next that $2\lambda a_{k'} < (1/2) - \hat{\mu}_L$ and $2\lambda a_k \geq (1/2) - \hat{\mu}_L$. Then an officeholder with ability a_k will produce utility in the second period that is equal to

$$-\left|\frac{1}{2} - \mu_L\right| + \lambda a_k,$$

which is larger than or equal to $-\lambda a_k$. According to Proposition 2, an officeholder with ability $a_{k'}$ will either generate utility in the second period that is equal to

$$-\left|\frac{1}{2} - \mu_L\right| + \lambda a_{k'}$$

or

$$-\left|\frac{1}{2} - 2\lambda a_{k'} - \frac{1}{2}\right| + \lambda a_{k'} = -\lambda a_{k'}.$$

Suppose finally that $2\lambda a_{k'} < (1/2) - \hat{\mu}_L$, and $2\lambda a_k < (1/2) - \hat{\mu}_L$. We distinguish three groups. First, officeholders with $\mu_L < (1/2) - 2\lambda a_k$ will produce utility $-\lambda a_k$ in the second period if they have ability a_k and $-\lambda a_{k'}$ if they have ability $a_{k'}$, because they choose ideological policies $(1/2) - 2\lambda a_k$ and $(1/2) - 2\lambda a_{k'}$, respectively. Second, the utility of the median voter from officeholders in the second period with $\mu_L \in \left[(1/2) - 2\lambda a_k, (1/2) - 2\lambda a_{k'}\right)$ is larger than $-\lambda a_k$ for incumbents with ability a_k and equal to $-\lambda a_{k'}$ for incumbents with ability $a_{k'}$. Third, officeholders with $\mu_L \geq (1/2) - 2\lambda a_{k'}$ and ability a_k will produce an additional utility gain $\lambda\left(a_k - a_{k'}\right)$ for the median voter in the second period, compared to an officeholder with ability $a_{k'}$, because both choose the same ideological policy.

Over both periods, depending on the realization of the ideal ideological point of the officeholder, the median voter's utility from having officeholder k in power in subsequent periods is equal or strictly higher than that of having officeholder k' in power. Taking the expectation over all possible realizations of ideological positions of officeholders proves the Corollary. ∎

PROPOSITION 10: *Suppose that a left-wing officeholder k with ability* $a_k \geq 0$ *has been reelected at the end of period* 1 *for a second term in period* 2, *with a vote share* s_1 ($s_1 \geq 1/2$). *There exists, then, a unique equilibrium continuation with the properties*

Case A: $(s_1 > \hat{\mu}_R)$[31]

(i) *If* $a_k < \left(1/(2\lambda)\right)\left(2\hat{\mu}_R - 1\right)$, *the officeholder is deselected. He selects* $i_{k2} = \mu_L$.

(ii) *If* $a_k \geq \left(1/(2\lambda)\right)\left(2\hat{\mu}_R - 1\right)$, *the officeholder is reelected and chooses the following policy*:

(α) *If* $2\lambda a_k \geq 3\hat{\mu}_R - (3/2)$, *the officeholder chooses* $i_{k2} = \mu_L$.

(β) *If* $2\lambda a_k < 3\hat{\mu}_R - (3/2)$, *the officeholder chooses*

$$i_{k2} = \begin{cases} \mu_L & \text{if } \mu_L \geq \mu_L^{cA} \\ \mu_L^{cA} & \text{if } \mu_L < \mu_L^{cA}, \end{cases} \tag{26}$$

with $\mu_L^{cA} := 2\hat{\mu}_R - (1/2) - 2\lambda a_k$.

[31] Since $\hat{\mu}_R$ is in the interval $[0,1]$, we can directly relate the vote share s_1 to the political position.

The equilibrium beliefs are given by

$$(27) \qquad \mu_{L2}^{e}(i_{k2}) = \begin{cases} i_{k2} & \text{if } i_{k2} > \mu_L^{cA} \\ \frac{\mu_L^{cA} + \hat{\mu}_L}{2} & \text{if } i_{k2} = \mu_L^{cA}. \\ \frac{i_{k2} + \hat{\mu}_L}{2} & \text{if } i_{k2} < \mu_L^{cA} \end{cases}$$

***Case B*:** $(1/2 \leq s_1 \leq \hat{\mu}_R)$

(iii) *If* $a_k < \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L + \frac{1}{2}}{2\lambda}$, *the officeholder is deselected where* $D(s_1) := -2s_1^2 + 4s_1\hat{\mu}_R - (1/4) - \hat{\mu}_R^2$.

(iv) *If* $a_k \geq \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L + \frac{1}{2}}{2\lambda}$, *the officeholder is reelected and chooses one of the following policies*:

(γ) *If* $a_k \geq \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L}{\lambda}$, *the officeholder chooses* $i_{k2} = \mu_L$.

(δ) *If* $a_k < \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L}{\lambda}$, *the officeholder chooses*

$$(28) \qquad i_{k2} = \begin{cases} \mu_L & \text{if } \mu_L \geq \mu_L^{cB} \\ \mu_L^{cB} & \text{if } \mu_L < \mu_L^{cB}, \end{cases}$$

with $\mu_L^{cB} := \frac{2D(s_1)}{2\hat{\mu}_R - 1} - 2\lambda a_k - \hat{\mu}_L$.

The equilibrium beliefs are

$$(29) \qquad \mu_{L2}^{e}(i_{k2}) = \begin{cases} i_{k2} & \text{if } i_{k2} > \mu_L^{cB} \\ \frac{\mu_L^{cB} + \hat{\mu}_L}{2} & \text{if } i_{k2} = \mu_L^{cB}. \\ \frac{i_{k2} + \hat{\mu}_L}{2} & \text{if } i_{k2} < \mu_L^{cB} \end{cases}$$

PROOF OF PROPOSITION 10:

Step 1: We show that the strategies and beliefs described in the proposition constitute an equilibrium. We can follow the steps in the proof of Proposition 2, but need to take into account that the officeholder needs a higher vote share to be reelected. The officeholder is reelected at the end of the period if and only if

$$(30) \qquad \lambda a_k - s_1 + \mu_{L2}^{e}(i_{k2}) \geq -E[|s_1 - \mu_R|],$$

where s_1 is the ideal point of the critical voter, who has to be at least indifferent between the officeholder and the challenger. The right-hand side is the expected utility from electing a right-wing challenger.

Step 2: We calculate $E\big[|s_1 - \mu_R|\big]$. We have to distinguish two cases.
If $s_1 > \hat{\mu}_R$, we immediately obtain

$$E\big[|s_1 - \mu_R|\big] = s_1 - \frac{\hat{\mu}_R + \frac{1}{2}}{2}.$$

If $1/2 \leq s_1 \leq \hat{\mu}_R$, the calculation of the expected utility losses is more subtle and is given by

$$E\big[|s_1 - \mu_R|\big] = \frac{s_1 - \frac{1}{2}}{\hat{\mu}_R - \frac{1}{2}} \cdot \left(s_1 - \frac{s_1 + \frac{1}{2}}{2}\right) + \frac{\hat{\mu}_R - s_1}{\hat{\mu}_R - \frac{1}{2}} \cdot \left(\frac{\hat{\mu}_R + s_1}{2} - s_1\right)$$

$$= \frac{1}{2\hat{\mu}_R - 1} \cdot \left(\left(s_1 - \frac{1}{2}\right)^2 + (\hat{\mu}_R - s_1)^2\right),$$

which yields

$$E\big[|s_1 - \mu_R|\big] = \frac{1}{2\hat{\mu}_R - 1} \cdot \left(2s_1^2 - 2s_1\left(\hat{\mu}_R + \frac{1}{2}\right) + \hat{\mu}_R^2 + \frac{1}{4}\right).$$

Step 3: We first deal with Case A, when $s_1 > \hat{\mu}_R$.
In this case, inequality (30) becomes

$$\lambda a_k + \mu_{L2}^e(i_{k2}) \geq \frac{\hat{\mu}_R + \frac{1}{2}}{2}. \tag{31}$$

For the belief $\mu_{L2}^e(i_{k2}) = \big(\hat{\mu}_L + (1/2)\big)/2$ and with $\hat{\mu}_L = 1 - \hat{\mu}_R$, we obtain $2\lambda a_k \geq 2\hat{\mu}_R - 1$. Thus, for $a_k < \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$, the incumbent will be deselected, as he cannot induce higher beliefs than $\big(\hat{\mu}_L + (1/2)\big)/2$ through his policy choice because all types of officeholders would choose such a position with this property if it existed.[32]

For $2\lambda a_k \geq 2\hat{\mu}_R - 1$, officeholders will choose positions such that they ensure reelection. To calculate such positions, we fix a_k and distinguish two cases:

- If $\lambda a_k + \hat{\mu}_L \geq \big(\hat{\mu}_R + (1/2)\big)/2$, or equivalently $2\lambda a_k \geq 3\hat{\mu}_R - (3/2)$, then all officeholders will be reelected if they choose their preferred policy—and thus all officeholders will choose $i_{k2} = \mu_L$.
- If $2\lambda a_k < 3\hat{\mu}_R - (3/2)$ (but $2\lambda a_k \geq 2\hat{\mu}_R - 1$ holds), officeholders below some threshold, say μ_L^{cA}, cannot afford to choose $i_{k2} = \mu_L$ while securing reelection.

[32] The actual equilibrium policy choices of such candidates are subtle. However, these choices will not matter for our results.

These officeholders are reelected if

$$\lambda a_k + \frac{\mu_L^{cA} + \hat{\mu}_L}{2} = \frac{\hat{\mu}_R + \frac{1}{2}}{2},$$

which yields

$$\mu_L^{cA} = 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k.$$

Hence, we obtain:

- If $a_k < \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$, the officeholder is deselected.
- If $a_k \geq \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$, the officeholder is reelected.
  - If $2\lambda a_k \geq 3\hat{\mu}_R - (3/2)$, officeholders choose $i_{k2} = \mu_L$.
  - If $2\lambda a_k < 3\hat{\mu}_R - (3/2)$, officeholders choose

$$i_{k2} = \begin{cases} \mu_L & \text{if } \mu_L \geq 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k \\ 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k & \text{if } \mu_L < 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k \end{cases}.$$

The equilibrium beliefs are given by

$$\mu_{L2}^e(i_{k2}) = \begin{cases} i_{k2} & \text{if } i_{k2} > 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k \\ \frac{2\hat{\mu}_R - 2\lambda a_k - \frac{1}{2} + \hat{\mu}_L}{2} & \text{if } i_{k2} = 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k. \\ \frac{i_{k2} + \hat{\mu}_L}{2} & \text{if } i_{k2} < 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k \end{cases}$$

Step 4: We next deal with Case B, when $1/2 \leq s_1 \leq \hat{\mu}_R$.
In this case, inequality (30) becomes

$$\lambda a_k - s_1 + \mu_{L2}^e(i_{k2}) \geq -\frac{1}{2\hat{\mu}_R - 1} \cdot \left(\left(s_1 - \frac{1}{2}\right)^2 + \left(\hat{\mu}_R - s_1\right)^2\right) \tag{32}$$

$$= -\frac{1}{2\hat{\mu}_R - 1} \cdot \left(2s_1^2 - 2s_1\left(\hat{\mu}_R + \frac{1}{2}\right) + \hat{\mu}_R^2 + \frac{1}{4}\right), \tag{33}$$

which yields

$$\lambda a_k\big(2\hat{\mu}_R - 1\big) + \mu_{L2}^e(i_{k2})\big(2\hat{\mu}_R - 1\big) \geq -2s_1^2 + 4s_1\hat{\mu}_R - \hat{\mu}_R^2 - \frac{1}{4}.$$

We use the abbreviation

$$D(s_1) := -2s_1^2 + 4s_1\hat{\mu}_R - \hat{\mu}_R^2 - \frac{1}{4}$$

and again distinguish three cases:

- If $a_k < \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L + \frac{1}{2}}{2\lambda}$, the officeholder will be deselected.
- If $a_k > \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L}{\lambda}$, the officeholder will be reelected if he chooses his preferred policy, no matter where his ideal policy is located.
- If a_k is between the above boundaries, the officeholder will either pool at the same policy choice or will choose their ideological position. More specifically, set μ_L^{cB} such that

$$\lambda a_k(2\hat{\mu}_R - 1) + \frac{\mu_L^{cB} + \hat{\mu}_L}{2}(2\hat{\mu}_R - 1) = D(s_1)$$

and thus

$$\mu_L^{cB} = \frac{2D(s_1)}{2\hat{\mu}_R - 1} - 2\lambda a_k - \hat{\mu}_L,$$

which yields the optimal choices

$$i_{k2} = \begin{cases} \mu_L & \text{if } \mu_L > \mu_L^{cB} \\ \mu_L^{cB} & \text{if } \mu_L \leq \mu_L^{cB}. \end{cases}$$

The beliefs are determined accordingly. ∎

PROOF OF PROPOSITION 6:

We first note that the median voter $i = 1/2$ is still decisive at the end of period 1 because the officeholder has received 50 percent of the votes for his first term. However, the future policies depend on the vote share a reelected officeholder receives at the end of period 1, which, in turn, has an impact on the voting behavior at the end of period 1. To analyze this situation, we organize the proof in several steps.

Step 1: In order to obtain a vote share of $s_1 \geq 1/2$, voter $i = s_1$ must be indifferent between reelecting the officeholder and electing a right-wing challenger. Suppose that the officeholder has ability a_k and that voters expect him to receive a vote share of s_1, with $1/2 \leq s_1 \leq 1$. Then, using Proposition 10, voter s_1's expected utility from reelecting the officeholder, denoted by V_{s_1}, can be calculated. This will be done for the various cases below. Deselecting the current officeholder and electing a right-wing challenger yields

$$V'_{s_1} = -\left(s_1 - \frac{1}{2}\right) - \frac{1}{2}\left(E\left[|s_1 - \mu_R|\right]\right) - \frac{1}{2}\left|s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right| + \lambda\frac{A}{4}, \tag{34}$$

recognizing that a newly elected officeholder k' will choose $i_{k'2} = 1/2$ in the second period and will be deselected if and only if $a_{k'} < 0$. Any officeholder will select his preferred ideological position in the last period. He may either be a right-wing or a left-wing politician.

We next investigate whether and, if yes, under which circumstances the various cases of Proposition 10 can occur as subgames in equilibrium.

Step 2: We start with Case A $(s_1 > \hat{\mu}_R)$ and the subcase A(i) from Proposition 10.

We investigate whether such a constellation can occur in equilibrium at the end of period 1.

If $a_k < \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$, the officeholder is deselected after a possible second term. Hence, the expected utility of voter s_1 from reelecting the officeholder at the end of period 1 is given by

$$V_{s_1} = \lambda a_k - \left| s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2} \right| - \left| s_1 - \frac{\hat{\mu}_R + \frac{1}{2}}{2} \right|, \tag{35}$$

where we have used that the officeholder is definitely rejected at the end of the second period.

Deselecting the officeholder at the end of period 1 and electing a right-wing challenger yields

$$V'_{s_1} = -\left(s_1 - \frac{1}{2}\right) - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_R + \frac{1}{2}}{2}\right) - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) + \lambda\frac{A}{4}. \tag{36}$$

Hence, using $\hat{\mu}_R = 1 - \hat{\mu}_L$, $V_{s_1} \geq V'_{s_1}$ if and only if $\lambda a_k \geq \lambda(A/4)$. Hence, voter s_1 reelects the officeholder at the end of period 1 if his ability fulfills

$$a_k \geq \frac{A}{4}. \tag{37}$$

If (37) holds, but $a_k < \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$, the constellation A(i) can occur in equilibrium.

Step 3: We next investigate the Case A(ii)α) in which $2\lambda a_k \geq 3\hat{\mu}_R - (3/2)$. In this case, we obtain

$$V_{s_1} = \lambda a_k - \left| s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2} \right| + \lambda a_k - \left| s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2} \right|. \tag{38}$$

Hence, $V_{s_1} \geq V'_{s_1}$, where V'_{s_1} is given by (36), if and only if

$$2\lambda a_k \geq -2\left(\frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) + \frac{1}{2} + \frac{1}{2}\left(\frac{\hat{\mu}_R + \frac{1}{2}}{2}\right) + \frac{1}{2}\left(\frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) + \lambda\frac{A}{4}$$

$$\Leftrightarrow$$

$$2\lambda a_k \geq \frac{1}{4} - \frac{3}{4}\hat{\mu}_L + \frac{1}{4}\hat{\mu}_R + \lambda\frac{A}{4},$$

which yields

$$a_k \geq \frac{1}{2\lambda}\Big(\hat{\mu}_R - \frac{1}{2}\Big) + \frac{A}{8}. \tag{39}$$

Hence, if condition (39) holds, constellation Case A(ii)α) can occur in equilibrium if the conditions of this case hold.

Step 4: With the same approach we examine the Case A(ii)β) and thus $a_k \geq \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$ and $2\lambda a_k < 3\hat{\mu}_R - (3/2)$. Since there are two further subcases that occur in the subgame, the calculations are more cumbersome. For the current officeholder, we obtain:

$$\begin{aligned}
V_{s_1} &= \lambda a_k - \Pr\Big[\mu_L \geq 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k\Big]\left(s_1 - \frac{2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k + \frac{1}{2}}{2}\right) \\
&\quad - \Big(1 - \Pr\Big[\mu_L \geq 2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k\Big]\Big)\Big(s_1 - \Big(2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k\Big)\Big) \\
&\quad + \lambda a_k - \left(s_1 - \frac{\frac{1}{2} + \hat{\mu}_L}{2}\right) \\
&= 2\lambda a_k - \frac{\frac{1}{2} - \Big(2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k\Big)}{\frac{1}{2} - \hat{\mu}_L}\left(s_1 - \frac{2\hat{\mu}_R - 2\lambda a_k}{2}\right) \\
&\quad - \frac{2\hat{\mu}_R - \frac{1}{2} - 2\lambda a_k - \hat{\mu}_L}{\frac{1}{2} - \hat{\mu}_L}\Big(s_1 - 2\hat{\mu}_R + \frac{1}{2} + 2\lambda a_k\Big) - \left(s_1 - \frac{\frac{1}{2} + \hat{\mu}_L}{2}\right) \\
&= 2\lambda a_k - 2s_1 - \frac{1}{\hat{\mu}_R - \frac{1}{2}}\Big[\big(1 - 2\hat{\mu}_R + 2\lambda a_k\big)\big(-\hat{\mu}_R + \lambda a_k\big) \\
&\qquad + \Big(3\hat{\mu}_R - \frac{3}{2} - 2\lambda a_k\Big)\Big(-2\hat{\mu}_R + \frac{1}{2} + 2\lambda a_k\Big)\Big] + \frac{\frac{3}{2} - \hat{\mu}_R}{2} \\
&= 2\lambda a_k - 2s_1 - \frac{1}{\hat{\mu}_R - \frac{1}{2}}\Big[\lambda a_k\big(6\hat{\mu}_R - 3 - 2\lambda a_k\big) + \Big(\frac{7}{2} - 4\hat{\mu}_R\Big)\hat{\mu}_R - \frac{3}{4}\Big] \\
&\quad + \frac{\frac{3}{2} - \hat{\mu}_R}{2}.
\end{aligned}$$

Hence, $V_{s_1} \geq V'_{s_1}$ if and only if

$$2\lambda a_k - 2s_1 - \frac{1}{\hat{\mu}_R - \frac{1}{2}}\Big[\lambda a_k\big(6\hat{\mu}_R - 3 - 2\lambda a_k\big) + \Big(\frac{7}{2} - 4\hat{\mu}_R\Big)\hat{\mu}_R - \frac{3}{4}\Big] + \frac{\frac{3}{2} - \hat{\mu}_R}{2}$$

$$\geq -\Big(s_1 - \frac{1}{2}\Big) - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_R + \frac{1}{2}}{2}\right) - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) + \lambda\frac{A}{4}$$

$$\Leftrightarrow\ 2(\lambda a_k)^2 - 2\lambda a_k(2\hat{\mu}_R - 1) - \Big(\hat{\mu}_R - \frac{1}{2}\Big)\lambda\frac{A}{4} + \frac{7}{8}(2\hat{\mu}_R - 1)^2 \ \geq\ 0,$$

which yields

$$a_k \ \geq\ \frac{(2\hat{\mu}_R - 1) + \sqrt{-\frac{3}{4}(2\hat{\mu}_R - 1)^2 + (2\hat{\mu}_R - 1)\lambda\frac{A}{4}}}{2\lambda}.$$

Since we are in Case A(ii)β), we have $a_k \ \geq\ \big(1/(2\lambda)\big)\big(2\hat{\mu}_R - 1\big)$ and thus, the second possible solution is irrelevant.[33]

Step 5: We next turn to Case B and start with the subcase (iii) of Proposition 10, which requires $a_k \ <\ \frac{D(s_1)}{\lambda(2\hat{\mu}_R - 1)} - \frac{\hat{\mu}_L + \frac{1}{2}}{2\lambda}$.

In this case, the officeholder—if he was reelected at the end of period 1—would be deselected at the end of period 2. Hence,

$$V_{s_1} \ =\ \lambda a_k - \left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) - E\big[|s_1 - \mu_R|\big],$$

while according to (34),

$$V'_{s_1} \ =\ -\Big(s_1 - \frac{1}{2}\Big) - \frac{1}{2}\big(E\big[|s_1 - \mu_R|\big]\big) - \frac{1}{2}\left|s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right| + \lambda\frac{A}{4}.$$

The term $E\big[|s_1 - \mu_R|\big]$ was calculated in the proof of Proposition 10 (Step 2) and is given by

$$E\big[|s_1 - \mu_R|\big] \ =\ \frac{1}{2\hat{\mu}_R - 1}\cdot\Big(2s_1^2 - 2s_1\Big(\hat{\mu}_R + \frac{1}{2}\Big) + \hat{\mu}_R^2 + \frac{1}{4}\Big).$$

Hence, $V_{s_1} \ \geq\ V'_{s_1}$ if and only if

$$\lambda a_k - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) - \frac{1}{2(2\hat{\mu}_R - 1)}\cdot\Big(2s_1^2 - 2s_1\Big(\hat{\mu}_R + \frac{1}{2}\Big) + \hat{\mu}_R^2 + \frac{1}{4}\Big)$$
$$+ \Big(s_1 - \frac{1}{2}\Big) - \lambda\frac{A}{4} \ \geq\ 0$$

[33] Moreover, there may not exist a solution in which voter s_1 is indifferent between the candidates. In such cases, the constellation cannot occur in equilibrium.

$$\Leftrightarrow$$

$$-\frac{1}{2\hat{\mu}_R - 1}s_1^2 + s_1\left(\frac{\hat{\mu}_R}{\hat{\mu}_R - \frac{1}{2}}\right) + \lambda a_k - \lambda\frac{A}{4} - \frac{1}{4}\left(\hat{\mu}_R + \frac{1}{2}\right) - \frac{\hat{\mu}_R^2 + \frac{1}{4}}{2(2\hat{\mu}_R - 1)} \geq 0$$

$$\Leftrightarrow$$

$$-s_1^2 + 2\hat{\mu}_R s_1 + (2\hat{\mu}_R - 1)\left[\lambda a_k - \lambda\frac{A}{4} - \frac{1}{4}\left(\hat{\mu}_R + \frac{1}{2}\right)\right] - \frac{1}{2}\left(\hat{\mu}_R^2 + \frac{1}{4}\right) \geq 0,$$

which yields

$$a_k \geq \frac{s_1^2 - 2\hat{\mu}_R s_1 + \frac{1}{2}\left(\hat{\mu}_R^2 + \frac{1}{4}\right)}{\lambda(2\hat{\mu}_R - 1)} + \frac{A}{4} + \frac{1}{4\lambda}\left(\hat{\mu}_R + \frac{1}{2}\right).$$

With $D(s_1) = -2s_1^2 + 4\hat{\mu}_R s_1 - \left(\hat{\mu}_R^2 + \frac{1}{4}\right)$, we obtain

$$a_k \geq \frac{-D(s_1)}{2\lambda(2\hat{\mu}_R - 1)} + \frac{A}{4} + \frac{1}{4\lambda}\left(\hat{\mu}_R + \frac{1}{2}\right). \tag{40}$$

If condition (40) and the conditions for Case B(iii) hold, we obtain an equilibrium constellation.

We also note that for a_k, which fulfills all conditions of the scenario in this step, we can solve for the vote share s_1 the officeholder obtains. We use the previous condition

$$-s_1^2 + 2\hat{\mu}_R s_1 + (2\hat{\mu}_R - 1)\left[\lambda a_k - \lambda\frac{A}{4} - \frac{1}{4}\left(\hat{\mu}_R + \frac{1}{2}\right)\right] - \frac{1}{2}\left(\hat{\mu}_R^2 + \frac{1}{4}\right) = 0,$$

which yields

$$-2s_1^2 + 4\hat{\mu}_R s_1 - \left(\hat{\mu}_R^2 + \frac{1}{4}\right) + \left(\hat{\mu}_R - \frac{1}{2}\right)\left(4\lambda a_k - \lambda A - \hat{\mu}_R - \frac{1}{2}\right) = 0$$

and

$$s_1^* = \hat{\mu}_R - \frac{1}{4}\sqrt{16\hat{\mu}_R^2 - 8\left(\hat{\mu}_R^2 + \frac{1}{4}\right) + 8\left(\hat{\mu}_R - \frac{1}{2}\right)\left(4\lambda a_k - \lambda A - \hat{\mu}_R - \frac{1}{2}\right)}$$

$$= \hat{\mu}_R - \sqrt{\left(\lambda a_k - \frac{\lambda A}{4}\right)(2\hat{\mu}_R - 1)}.$$

Step 6: We continue with Case B(iv)γ) of Proposition 10.
In this case, expected utilities are given by

$$V_{s_1} = 2\lambda a_k - 2\left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right),$$

$$V'_{s_1} = -\left(s_1 - \frac{1}{2}\right) - \frac{1}{2}\left(E\left[|s_1 - \mu_R|\right]\right) - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) + \lambda\frac{A}{4},$$

with

$$E\left[|s_1 - \mu_R|\right] = \frac{1}{2\hat{\mu}_R - 1}\cdot\left(2s_1^2 - 2s_1\left(\hat{\mu}_R + \frac{1}{2}\right) + \hat{\mu}_R^2 + \frac{1}{4}\right).$$

Thus, $V_{s_1} \geq V'_{s_1}$ if and only if

$$2\lambda a_k - \frac{3}{2}s_1 + \frac{9}{8} - \frac{3}{4}\hat{\mu}_R + s_1 - \frac{1}{2}$$
$$+ \frac{1}{2(2\hat{\mu}_R - 1)}\cdot\left(2s_1^2 - 2s_1\left(\hat{\mu}_R + \frac{1}{2}\right) + \hat{\mu}_R^2 + \frac{1}{4}\right) - \lambda\frac{A}{4} \geq 0$$

$$\Leftrightarrow$$

$$2\lambda a_k + \frac{5}{8} - \frac{3}{4}\hat{\mu}_R - \lambda\frac{A}{4} + \frac{1}{2\hat{\mu}_R - 1}\cdot\left(s_1^2 - 2\hat{\mu}_R s_1 + \frac{1}{2}\left(\hat{\mu}_R^2 + \frac{1}{4}\right)\right) \geq 0,$$

which yields

$$a_k \geq \frac{D(s_1)}{4\lambda(2\hat{\mu}_R - 1)} + \frac{A}{8} + \frac{3\hat{\mu}_R - \frac{5}{2}}{8\lambda}, \tag{41}$$

where $D(s_1)$ is again $D(s_1) = -2s_1^2 + 4\hat{\mu}_R s_1 - \hat{\mu}_R^2 - (1/4)$. Hence, under the condition of Case B(iv)γ) and condition (41), we obtain an equilibrium where the officeholder is reelected at the end of period 1 with the behavior described in the corresponding case in Proposition 10. One can also solve for the equilibrium vote share the incumbent will receive. From

$$2s_1^2 - 4\hat{\mu}_R s_1 + \hat{\mu}_R^2 + \frac{1}{4} + \left(\hat{\mu}_R - \frac{1}{2}\right)\left(8\lambda a_k - \lambda A - 3\mu_R + \frac{5}{2}\right) = 0,$$

we obtain

$$s_1^* = \hat{\mu}_R - \frac{1}{4}\sqrt{16\hat{\mu}_R^2 - 8\left(\hat{\mu}_R^2 + \frac{1}{4} + \left(\hat{\mu}_R - \frac{1}{2}\right)\left(8\lambda a_k - \lambda A - 3\hat{\mu}_R + \frac{5}{2}\right)\right)}$$

$$= \hat{\mu}_R - \sqrt{(2\hat{\mu}_R - 1)\left(\hat{\mu}_R - \frac{1}{2} + \frac{\lambda A}{4} - 2\lambda a_k\right)}.$$

Step 7: Finally, we turn to the Case B(iv)δ).
We calculate the expected utilities as

$$V_{s_1} = \lambda a_k - \Pr\left[\mu_L \geq \mu_L^{cB}\right]\left(s_1 - \frac{\mu_L^{cB} + \frac{1}{2}}{2}\right) - \left(1 - \Pr\left[\mu_L \geq \mu_L^{cB}\right]\right)\left(s_1 - \mu_L^{cB}\right)$$
$$+ \lambda a_k - \left(s_1 - \frac{\frac{1}{2} + \hat{\mu}_L}{2}\right),$$

where $\Pr\left[\mu_L \geq \mu_L^{cB}\right] = \left((1/2) - \mu_L^{cB}\right)/\left((1/2) - \hat{\mu}_L\right)$.

$$V'_{s_1} = -\left(s_1 - \frac{1}{2}\right) - \frac{1}{2}\left(E\left[|s_1 - \mu_R|\right]\right) - \frac{1}{2}\left(s_1 - \frac{\hat{\mu}_L + \frac{1}{2}}{2}\right) + \lambda\frac{A}{4}.$$

Again, we have

$$E\left[|s_1 - \mu_R|\right] = \frac{1}{2\hat{\mu}_R - 1} \cdot \left(2s_1^2 - 2s_1\left(\hat{\mu}_R + \frac{1}{2}\right) + \hat{\mu}_R^2 + \frac{1}{4}\right)$$
$$= \frac{-D(s_1)}{2\hat{\mu}_R - 1} + s_1.$$

Hence, $V_{s_1} \geq V'_{s_1}$ if and only if

$$(42) \qquad \frac{32D(s_1)^2 - 4D(s_1)(15 + 16\lambda a_k - 14\hat{\mu}_R)(2\hat{\mu}_R - 1)}{8(2\hat{\mu}_R - 1)^3} - \frac{\lambda A}{4}$$
$$+ \frac{27 + 16\lambda a_k(3 - 2\hat{\mu}_R) + 32(\lambda a_k)^2 - 48\hat{\mu}_R + 20\hat{\mu}_R^2}{8(2\hat{\mu}_R - 1)} \geq 0.$$

From this inequality, we can obtain the critical value of a_k by solving the corresponding quadratic equation.[34] Moreover, we denote the left-hand side of (42), viewed as a function of s_1, by $\alpha(s_1)$.

Step 8: Since the officeholder has received 50 percent of the votes for his election to his first term, the median voter is critical for the reelection bid. We next claim that an officeholder is reelected if and only if $a_k \geq \hat{a}^{SR}$, where $\hat{a}^{SR}$ is the solution of $\alpha(1/2) = 0$. To prove this claim, we first observe that Case B(ii)δ) is the relevant case when we explore the threshold for ability to yield reelection at the end of period 1. The reason is threefold. First, if an officeholder is reelected with ability $\hat{a}^{SR}$, all officeholders with $a \geq \hat{a}^{SR}$ will be reelected, as the median voter and all left-wing-oriented citizens prefer such a candidate over the alternative path with a new challenger. Specifically, V_{s_1} is strictly increasing in a_k, while V'_{s_1} is independent of a_k. Second, if the officeholder is reelected at the end of period 1, he can also secure reelection at the end of period 2, since he can choose the median voter's position in the second term again. The minimal vote share for reelection is $s_1 = 1/2$ and thus Case B applies. Third, from the two remaining subcases, Case B(ii)δ) applies, as it concerns the lowest ability values for which reelection is possible.

Second, we observe that if an officeholder with ability a_k is reelected with vote share $s_1 > 1/2$, an officeholder with ability $a_k - \varepsilon$ for $\varepsilon > 0$ arbitrary small, is also reelected. This follows the continuity of V_{s_1} and V'_{s_1} with respect to a_k and s_1

[34] The cumbersome expression is available upon request.

and the monotonicity properties. Hence, the minimal ability at which a candidate is reelected is given by $\hat{a}^{SR}$.

We note that

$$\alpha\left(\frac{1}{2}\right) = -\frac{\lambda A}{4} + \frac{4\left(\lambda a_k\right)^2}{2\hat{\mu}_R - 1}.$$

Therefore $\alpha(1/2) = 0$ yields

$$\hat{a}^{SR} = \frac{1}{2}\sqrt{\frac{A}{2\lambda}\left(\frac{1}{2} - \hat{\mu}_L\right)}.$$

Step 9: We verify whether a candidate wants to deviate from the centrist policy choice in period 1. Suppose an officeholder k with ideal point μ_L chooses an ideological policy $i_{k1} \in [\mu_L, 1/2)$. Then, he will partly reveal his ideological orientation, which is the expected value of $(i_{k1} + \hat{\mu}_L)/2$, as these are the out-of-equilibrium beliefs. On the one hand, this yields an immediate utility gain $i_{k1} - (1/2)$. On the other hand, reelection chances are reduced from

$$\Pr\left[a_k > \hat{a}^{SR}\right] = \frac{1}{2} - \frac{\hat{a}^{SR}}{2A}$$

to

$$\Pr\left[a_k > \tilde{a}\right] = \frac{1}{2} - \frac{\tilde{a}}{2A},$$

where $\tilde{a}$ is the new ability threshold above which the deviating officeholder will be reelected. This new ability threshold increases proportionally to i_{k1} and can be calculated from the preceding steps as

$$\tilde{a} = \frac{1}{2}\sqrt{\frac{A}{2\lambda}\left(\frac{1}{2} - \hat{\mu}_L + \left(\frac{1}{2} - i_{k1}\right)\right)}.$$

The officeholder suffers a utility loss from deselection, which consists of fewer expected non-monetary benefits b from holding power, less favorable ideological policy choices and public project levels (since $\tilde{a} > \hat{a}^{SR} > 0$) delivered by a new officeholder. The total utility loss is larger than

$$2b\left(\Pr\left[a_k > \hat{a}^{SR}\right] - \Pr\left[a_k > \tilde{a}\right]\right). \tag{43}$$

Equating (43) with the utility gain $i_{k1} - (1/2)$ yields a value for b, denoted by $\bar{b}$. A deviation from $i_{k1} = 1/2$ is thus not profitable if $b \geq \bar{b}$.

We note that a tighter lower bound $\bar{b}$ can be found by including the utility losses that arise from less attractive ideological policy choices by the new officeholder who will replace the incumbent in case of deselection. This tighter bound can be calculated by weighting all possible policy and ability differences that occur when the current officeholder is deselected, as derived in the previous steps.[35]

[35] The calculations are extremely cumbersome and available upon request.

Step 10: It is useful to verify that strategies and beliefs do indeed constitute an equilibrium. While beliefs have already been dealt with in Proposition 10, the following considerations are relevant.

If $a_k = \hat{a}^{SR}$ in Case B(ii)δ), the median voter is just indifferent between the incumbent and the challenger, anticipating that the former will receive a vote share of $1/2$. Hence, an incumbent with $a_k = \hat{a}^{SR}$ is reelected.

If $a_k > \hat{a}^{SR}$, the equilibrium value for s_1 is larger than 50 percent. Suppose that all voters with $i \leq s_1$ support the reelection bid. Then, voters anticipate that the reelected incumbent will face the vote threshold $s_1(a_k)$ under the SR-rule in the next reelection and will choose the policies accordingly (see Proposition 10). Then, indeed, voters with $i > s_1$ are strictly worse off by reelecting the officeholder, while voters with $i < s_1$ benefit. By construction, voter $i = s_1$ is indifferent between reelecting the incumbent and electing a new right-wing candidate. Hence, the voting behaviors are indeed best responses.

Suppose that $a_k < \hat{a}^{SR}$ and $s_1 > 1/2$. However, there is no $s_1 > 1/2$ at which the incumbent could gain the support of s_1 voters and at which these voters would anticipate that he wins with a share of s_1 votes. Voter s_1 would need to be indifferent between reelecting the incumbent and electing the challenger, which is impossible because $a_k < \hat{a}^{SR}$.

To sum up, the strategies and beliefs constructed in the proof constitute an equilibrium. ∎

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