

Connecting Disconnected Financial Markets?[†]

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In most financial markets, securities are traded in isolation. Such a disconnected market design can be inefficient if agents trade more than one security. I assess welfare effects of connecting markets by allowing orders for one security to depend on prices of other securities. I show that everyone trades identical amounts under both market structures if and only if the clearing prices are perfectly correlated or all are price-takers. Prices in disconnected markets might allow strategic traders to extract higher rents from nonstrategic traders. In expectation, connected markets generate higher welfare, but all markets become efficient as they grow large. (JEL D44, D47, G10, H82)

Modern economies consist of markets with different structures. Traditionally, many markets were *disconnected*, in that only one good was sold or traded per transaction. Examples are markets for spectrum and mineral rights, oil and gas royalties, dairy products, and aquarian animals. Modern technology and higher computing power have made it feasible to connect such markets. The most prominent example is the incentive auction for US spectrum rights (Milgrom et al. 2012). Between 2016 and 2017, this gigantic combinatorial auction reallocated many different spectrum licenses across the country. In such a *connected* market, participants are allowed to make the demand for one good (here a particular license) contingent on the price of another good. In other words, agents are free to ask for packages of the goods. Other markets have been connected in a similar fashion.

Nonetheless, most financial markets remain disconnected. While it is possible to trade a German bond and a French one at the same time—sometimes even within the same platform—one typically cannot interlink the demand for these two bonds when markets are disconnected. One exception is the qualified contingent cross order type, launched by the International Securities Exchanges (now part of NASDAQ) in February 2011. In this market design, an exchange operator connects the central limit order books of two distinct markets: stocks and options. This and other examples demonstrate that it is technologically feasible to connect financial

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markets, yet it is uncommon.¹ Therefore traders are typically constrained in terms of how they can display their joint preferences for the goods. They cannot freely maximize their gains from trade, as it is not possible to ask for or offer security bundles—for instance, in the form of submitting contingent cross orders.

This paper provides a rationale for why financial markets remain disconnected and examines the welfare effects of connecting such markets in a framework in the tradition of Kyle (1989). In my model, strategic agents with normally distributed private types derive joint (quadratic) utility from owning two perfectly divisible goods. They trade with one another and with noise traders, who demand or supply a random amount of either good, in a *connected* or *disconnected* market.

The disconnected market consists of two separate uniform-price double auctions of a single good: in each auction, agents specify a price for each quantity they demand or supply, similar to packages or batches of infinitesimally small limit orders. The market clears at the price at which aggregate demand meets aggregate supply, and each agent buys or sells what he demanded or offered at this price. When the market is connected, an agent is allowed to bid for bundles. More precisely, the rules of the standard uniform-price double auction are extended to allow the demand for one good to depend on the price of the other good. Notably, the framework can easily be modified to fit other markets, including nonfinancial markets in which all market participants either sell or buy but do not trade.

I provide two sets of results. Both rely on a comparison of the unique symmetric linear Bayesian Nash equilibrium across market structures.² The first set consists of irrelevance theorems. They specify the conditions under which the disconnected market achieves the same allocation as the connected market, even though agents are—by the market design—constrained regarding how they can display their joint preferences. I show that agents trade the exact same amounts under both market structures *if and only if* the (endogenous) market-clearing prices are perfectly (positively or negatively) correlated or the market grows large, so that all agents become price-takers. In small markets in which agents are strategic, the corresponding clearing prices may differ because of strategic bid shading. This implies that assets may trade at different prices, even though markets are equally allocative efficient. Under imperfect price correlation, traded quantities of small markets differ *ex post* but still coincide *in expectation*. Similarly, the expected clearing prices exhibit the same features as realized clearing prices under perfect correlation.

¹In addition, other financial services corporations and brokerages, such as Fidelity Investments and Interactive Brokers, allow for contingent orders for which execution may depend on the execution and/or price of another security. Another exception is the product-mix auction through which the Bank of England performs its index long-term repo operations (ILTRs). Proposed by Klemperer (2010), it allows participants to bid on multiple differentiated goods (product varieties) in a single auction, with each bid specifying a price and quantity for a variety.

²Notably, I do not restrict attention to linear *ex post* optimal equilibria, in contrast to most related literature following Klemperer and Meyer (1989) and Kyle (1989). *Ex post* optimal equilibria are invariant to the distribution of random factors, which implies that correlations across endogenous clearing prices never matter for the investment decisions of traders. As this is at odds with what seems to matter in real life, in which cross-market correlations are observed carefully and seem to be crucial determinants for successful portfolio choices, I do not require *ex post* optimality.

The second set of results assesses who wins and who loses when connecting disconnected markets and compares how much welfare is generated under either market structure. When markets are small and prices are perfectly correlated, I show that strategic agents often obtain a higher total surplus (utility – payment) in the disconnected market than they do in the connected market. The reason is that they can extract higher rents from noise traders. This result carries over to the more general case in which prices may be imperfectly correlated.

By means of stylized examples, I argue that connected markets must not necessarily be more efficient than disconnected markets because of two opposing effects. On the one hand, connected markets are more efficient, as agents can freely maximize gains from trade; on the other, a more expressive bidding language—that is, being able to make cross-price contingent orders—may give at least some strategic agents more options to manipulate the clearing price and thereby distort the allocation. When the correlation of clearing prices is sufficiently mild, the second effect never dominates. I show that the connected market Pareto dominates the disconnected market, in that all strategic agents realize higher utility at market clearing (ex post). Further, connected markets prove to generate higher welfare in expectation. As the market grows large, any difference washes out, and either market structure achieves the efficient allocation (ex post).

My findings have important implications for the design of real markets. The irrelevance theorems identify key factors that matter when evaluating whether to connect disconnected markets: the (endogenous) correlation of prices, the size of the market, and whether traders are strategic or price-taking. More broadly, the theorems tell us what goes wrong when the conditions under which they hold are violated. Similar to other irrelevance statements, including the most influential ones (such as Modigliani and Miller 1958 or Coase 1960), those conditions are extreme (such as perfect markets or zero transaction costs). In combination with the welfare analysis, the theorems provide guidance on when it might be worthwhile to pay the costs of modifying existing market structures and when it is not.

Further, my main equivalence result provides a rationale for why some real-world markets that exhibit high price correlations remain disconnected. Here I give three examples. The first comes from the primary market of government bonds. Several countries, including the world's largest economies (the United States, China, Japan, Germany, France, India, Brazil, and Canada), issue bonds of different maturities in separate, parallel auctions rather than in a single combinatorial auction such as the product-mix auction. In the terminology of this paper, the market is classified as small and disconnected. Using data from the US primary market for T-bills, I illustrate that market-clearing prices for different maturities are essentially perfectly correlated ($\rho > 0.99$).

A second example is derivative contracts, which are often traded in different markets than their underlying assets. In particular, in the short run, the price for the derivative is typically highly positively/negatively correlated (e.g., put/call options).

The third and last example features identical assets that are traded in separate (“fragmented”) markets: equity and fixed income securities are traded in dozens of trading venues, none with dominating market shares. As my analysis predicts losses for strategic traders if one were to connect these markets, it points to the possibility

that large traders (such as dealers or large hedge or mutual funds) who have price impact—and therefore incentives to act strategically—might also have incentives to prevent changes in the existing, disconnected market structure.

My main methodological contribution belongs to the literature on multiunit auctions of perfectly divisible goods.³ Share auctions were introduced by Wilson (1979) for single-sided auctions and closely relate to Klemperer and Meyer (1989), Kyle (1989), Vives (2011), and Rostek and Weretka (2012). With few exceptions, this literature considers an auction in isolation, thereby neglecting possible interconnections across auction markets.⁴ I am, to the best of my knowledge, the first to derive an equilibrium in separate uniform-price auctions that offer related goods.

Similar to the related literature, my linear equilibrium relies on the assumption that all agents have a quadratic utility function and random variables are normally distributed. My necessary optimality condition for the Bayesian Nash equilibrium (BNE) of the disconnected market, in contrast, holds for a broad class of utilities and any random variables that have differentiable distribution functions. This enables me to explain the strategic incentives that lie behind the equilibrium more broadly. Moreover, it extends to the other most frequently used (sealed-bid) multiunit auction format: the pay-as-bid (also called discriminatory price) auction, as shown by Wittwer (2020). A discretized version of these necessary conditions is ideal for structurally estimating interdependencies in valuations of bidders in simultaneous multiunit auctions such as Treasury auctions (e.g., Allen, Kastl, and Wittwer 2020).

Related Literature.—My research topic broadly fits into the literature that compares the performance of decentralized or fragmented markets with that of centralized markets. Decentralized markets are often studied in search, bargaining, or network models, typically underlying different types of inefficiencies (e.g., Miao 2006, Elliott and Nava 2019, Elliott 2015). In formulating an auction model, I take a different approach from that of previous studies. I highlight a different aspect of fragmentation: demand for a good offered in one market that clears through a centralized mechanism cannot be made contingent on the price of another good. To avoid confusion, I call such a market “disconnected,” rather than “decentralized,” and refer to its counterpart as “connected.”

I am aware of only a few papers that highlight this aspect of market fragmentation. One exception is Zigrand (2005), which studies asset prices in a (rational) competitive equilibrium. As opposed to Zigrand (2005), I allow agents to behave strategically in order to provide insights for markets in which not all agents are price-takers. This is common in two new working papers that adapt frameworks that are close to mine: Rostek and Yoon (2020) analyzes a setting with multiple markets

³Similar to the frequent assumption in the literature on single-unit auctions that the set of available prices is dense, the assumption of perfect divisibility is a continuous approximation of a discrete set of quantities—which has long been recognized across economic disciplines as a valuable alternative when discrete problems are intractable (Woodward 2015). With imperfect divisibility of goods or buyers who can submit only a maximal amount of bids, the analysis becomes more complex due to discontinuities and rationing. This has been demonstrated by Hortaçsu and McAdams (2010) and Kastl (2011, 2012).

⁴A growing literature addresses dynamics across time. Examples include Vayanos (1999), Rostek and Weretka (2015), Sannikov and Skrzypacz (2016), Du and Zhu (2017b), Antill and Duffie (forthcoming), Rostek and Yoon (2019).

(called exchanges), while Chen and Duffie (2020) studies what happens as the number of (disconnected) markets that offer identical assets increases. In contrast, I fix the number of markets but allow assets to differ across markets.⁵

By letting disconnected markets clear via simultaneous multiunit auctions, my work relates to a growing literature put forward by computer scientists. Motivated by Bikhchandani (1999, 212), which warned that “simultaneous sealed bid auctions are likely to be inefficient under incomplete information,” this literature quantifies the efficiency of simultaneous auctions of heterogeneous goods by computing the “price of anarchy.” They “prove convergence to full efficiency without ever characterizing what these near-optimal equilibria look like” (Feldman et al. 2016, 2). I do not approximate and instead characterize allocations precisely. Two contributions are most related to my work. First, Syrgkanis and Tardos (2013) shows that m simultaneously run uniform-price auctions achieve “at least” $(e - 1)/(4e) \approx 0.158$ of the expected optimal effective welfare. Second, Feldman et al. (2016) proves that such auctions lead to a fully efficient allocation when the market grows large, which is in the spirit of my third irrelevance theorem.⁶

Malamud and Rostek’s (2017) findings are orthogonal to this literature. They demonstrate that decentralized markets might be more efficient than centralized markets, similar to Glode and Opp (2019). In Malamud and Rostek’s (2017) network model, strategic traders may participate in several markets, each defined by the subset of agents who trade there and the subset of assets traded. Their framework is similar to mine in that markets clear simultaneously via multiasset uniform-price double auctions. It differs in that agents are free to submit cross-price contingent demand schedules that depend on the prices of *all* exchanges they participate in. In my terminology, their market is (always) connected.

At the other extreme, Budish, Lee, and Shim (2019) focus on disconnected markets of identical goods. They introduce a theoretical model of (US) stock exchange competition to study whether exchanges have incentives to foster the adoption of more efficient market designs that eliminate an inefficient race for high-speed technologies. Their model differs significantly from mine, but some of our findings capture similar ideas. For example, they argue that exchanges have no incentive to break up existing market structures, because they would lose profit. In my setting, strategic traders have no incentives to foster the connection of markets.

Coming from many different directions and using a wide variety of techniques, most of these articles agree that welfare hinges on the market’s microstructure. My first irrelevance theorem goes against this broad consensus. Even though it is specific to particular applications, it is in the spirit of famous general theorems that tell us when “market structure” in different formats is irrelevant.⁷

⁵Formally, traders in any model have private information that must not be perfectly correlated across markets. This generates imperfectly correlated market-clearing prices (even in absence of noise traders)—a feature that is realistic for most real-world applications and complicates the derivation of the equilibrium.

⁶In Feldman et al. (2016), a fixed set of different goods that are divisible into a finite number of units is auctioned off to a set of bidders with combinatorial valuations over sets of units of different items. The market grows large as the number of players (who fail to arrive in the market with some positive probability) and the number of units of each good’s supply increase.

⁷Sah and Stiglitz (1987) and Dasgupta (1988) establish conditions under which the number of firms (= market structure) does not matter for technological innovation; Modigliani and Miller (1958) proves that the financial

The most recent equivalence result comes from Holmberg, Ruddell, and Willems (2019). In their model, two firms each produce two perfectly divisible goods. They participate in a uniform-price procurement auction by submitting a two-dimensional supply function for each good. With the option to submit cross-price contingent supply curves, their market classifies as connected in my terminology. They show that BNE allocation of the two goods and the equilibrium payoffs of the suppliers are invariant with respect to bundling. This result can be seen as an invariance result for what occurs *within* a connected market. My comparison goes across market structures.

The remainder of the paper is structured as follows: Section I sets up the model. Section II establishes equilibrium existence. Section III provides three irrelevance theorems. Section IV discusses welfare implications. Section V gives ideas for future research on disconnected markets, and Section VI concludes. All proofs are given in the Appendix. Throughout the paper, random variables are highlighted in **bold**.

I. Model

There are $n > 2$ agents who trade two perfectly divisible goods, indexed $m = 1, 2$, in a connected or disconnected market.⁸ The connected market is modeled as a multigood uniform-price double auction. The disconnected market consists of two separate single-good uniform-price double auctions that are run simultaneously.

Holding all other rules of the game fixed allows me to focus on the effect of connecting disconnected markets. If I were to compare the separate uniform-price auctions to some other combinatorial auction, I would no longer be able to separate the effect of centralization from those that arise from changes of other rules of the transaction.

Information Structure.—In either environment, the model allows for two sources of uncertainty. The first is idiosyncratic: each agent has private information. He draws a two-dimensional, private type $\bar{s}_i \equiv (s_{i,1} \ s_{i,2})$. It captures individual preferences and personal evaluations of risk or, if the agent is part of a large financial institution, individual orders from customers. It will determine the agent's true demand.⁹ The second is on the aggregate level. It comes from noise traders (as in Kyle 1989) or an auctioneer who may generate random exogenous supply or

structure of the firm (= market structure) does not necessarily matter for the creation of value; Weber (1983) shows that the realized price of any auction game that sells identical objects (= market structure) is the realized price of the previous auction; Vickery (1961) proves that some rules of the auction (= market structure) are irrelevant for the seller's expected revenue. Building on the revenue equivalence theorem, Biais (1993) then demonstrates that centralized and fragmented markets with risk-averse agents who compete for a single market order (= market structure) may give rise to the same expected ask (bid) price.

⁸With $n = 2$ agents, the nonexistence of equilibria has long been recognized in the literature when marginal utility is decreasing (e.g., Kyle 1989 from Ausubel et al. 2014, Du and Zhu 2017a).

⁹Here I break with the traditional view according to which the demand of financial securities is driven by common values, partially to keep the model tractable. While it is true that the price of a security stabilizes in the long run, building the basis for a common valuation among agents, it fluctuates a lot in the short run. In today's financial markets, which move at high speed, individual factors might therefore explain individual demand in the short run more adequately. Empirical evidence from the primary market of government bonds supports this view (Hortaçsu and Kastl 2012).

demand for each good $(\mathbf{Q}_1, \mathbf{Q}_2)$. For convenience, I will refer to these exogenous amounts as “supply.”

To achieve tractability, I need to impose distributional form assumptions. For one, types and supply are normally distributed. This is a common simplifying assumption in the related literature, as it allows one to solve for equilibria that are linear (e.g., Kyle 1989, Vives 2011, and Rostek and Wernetka 2012). Second, I consider symmetric distributions. More precisely, the two types of an agent $(s_{i,1}, s_{i,2})$, as well as the two total supply quantities $(\mathbf{Q}_1, \mathbf{Q}_2)$, share the same variance for both goods. This will create ex ante symmetric market conditions and lead to equilibria that are symmetric across goods (see Section V for further discussion). Finally, types are private and drawn independently *across agents* and supply. Notably, the two valuations $(s_{i,1}, s_{i,2})$ of the same agent i may be correlated. Summarizing, I assume

$$\begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{Q_1} \\ \mu_{Q_2} \end{pmatrix}, \sigma_Q^2 \begin{pmatrix} 1 & \rho_Q \\ \rho_Q & 1 \end{pmatrix}\right) \quad \text{and} \quad \begin{pmatrix} s_{i,1} \\ s_{i,2} \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{s_1} \\ \mu_{s_2} \end{pmatrix}, \sigma_s^2 \begin{pmatrix} 1 & \rho_s \\ \rho_s & 1 \end{pmatrix}\right),$$

with \bar{s}_i being drawn iid across i and $\mathbf{Q}_1, \mathbf{Q}_2$.

Relative to the related literature, the assumption that agents draw their private types independently from one another is perhaps the strongest assumption. It simplifies the algebra when solving for the equilibrium of a disconnected market and therefore allows for a tractable, formal comparison across market structures. The intuition that drives my results does not seem to rely on this assumption, as I will explain later on. Further, my main methodological contribution, which is to derive an equilibrium in the disconnected market (= parallel uniform-price double auctions), is not limited by the independence assumption. In online Appendix E, I illustrate how to modify my proof and derive an equilibrium in a setting in which types are correlated across agents (see Section V for further discussion). A similar approach could be taken to obtain an equilibrium when agents have interdependent values of a simple (linear) form such as $\alpha s_{m,i} + (1 - \alpha) \sum_{j \neq i} s_{m,i}$, where $0 \leq \alpha \leq 1$.

I focus my analysis on environments in which traders have at least some uncertainty in assuming $\sigma_s > 0$. Noise traders, instead, may be excluded from the market by setting $\sigma_Q = \mu_{Q_1} = \mu_{Q_2} = 0$. Alternatively, they might provide a known supply Q_1 and Q_2 of nonzero amounts. Strictly speaking, supply in this case is not drawn from the depicted normal distribution but is fixed at $Q_1 = \mu_{Q_1}$ and $Q_2 = \mu_{Q_2}$. As a benchmark, I choose the environment with random supply ($\sigma_Q > 0$) for my formal analysis.

Strategies.—All agents submit a pair of demand functions for each good:

- (1) $x_{i,m}(\cdot, \bar{s}_i) : \mathbb{R} \rightarrow \mathbb{R}$ for $m = 1, 2$ in the disconnected market;
- ($\bar{1}$) $\bar{x}_i(\cdot, \bar{s}_i) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ in the connected market.

These functions may be seen as packages of limit orders of infinitesimally small size, where perfect divisibility is a continuous approximation of a discrete set of quantities or distinct orders. They specify how much the agent is willing to

buy $q_m > 0$ or sell $q_m < 0$ and at what price/s. Their inverses are bidding schedules, denoted $b_{i,m}(\cdot, \vec{s}_i)$, $\vec{b}_i(\cdot, \vec{s}_i)$. They specify a price per quantity/ies and are assumed to be decreasing. To obtain mathematical tractability in the disconnected market, I further restrict attention to functions that are twice continuously differentiable and asymptotically linear.

DEFINITION 1: A differentiable function $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ is asymptotic linear if $\lim_{x \rightarrow +\infty} f'(x)$ and $\lim_{x \rightarrow -\infty} f'(x)$ are constants.

Importantly, the functions need not be linear in the interior of their domain. Since agents never win an infinite amount of the good with positive probability, this purely technical assumption will never affect the equilibrium. It ensures that the expected value of the agent’s total surplus at market clearing exists even if the agent deviates from the equilibrium. The restriction would not be needed if all random variables had bounded support, and it is similar in spirit to the more commonly made assumption of finite clearing prices.

To facilitate comparison across market structures, I will often refer to demands in the connected market for each good separately, denoting $\vec{x}_i(\vec{p}, \vec{s}_i) = (\vec{x}_{i,1}(p_1, p_2, \vec{s}_i) \vec{x}_{i,2}(p_2, p_1, \vec{s}_i))'$ and $\vec{b}_i(\vec{q}, \vec{s}_i) = (\vec{b}_{i,1}(q_1, q_2, \vec{s}_i) \vec{b}_{i,2}(q_2, q_1, \vec{s}_i))'$.

Market Clearing.—Once all agents have submitted their demands, both markets clear at the price/s at which aggregate supply meets aggregate demand.

For good 1:

$$(MC) \quad Q_1 = \sum_i x_{i,1}(p_1^c, \vec{s}_i) \quad \text{in the disconnected market;}$$

$$(\overline{MC}) \quad Q_1 = \sum_i \vec{x}_{i,1}(\vec{p}_1^c, \vec{p}_2^c, \vec{s}_i) \quad \text{in the connected market.}$$

Each agent then buys or sells what he asked for at this price, abbreviated by $q_{i,1}^c \equiv x_{i,1}(p_1^c, \vec{s}_i)$ and $\vec{q}_{i,1}^c \equiv \vec{x}_{i,1}(\vec{p}_1^c, \vec{p}_2^c, \vec{s}_i)$. He makes a total payment of $\sum_m p_m^c q_{i,m}^c$ in the disconnected market and $\sum_m \vec{p}_m^c \vec{q}_{i,m}^c$ in the connected market. Market-clearing prices that arise in equilibrium and the corresponding quantities will be denoted by p_m^* , $q_{i,m}^*$ and \vec{p}_m^* , $\vec{q}_{i,m}^*$.

If the aggregated demand at the clearing price is higher than the aggregate supply, then agents have to be rationed according to some tie-breaking rule. However, equilibrium demand functions will be strictly decreasing in quantity/ies so that all markets will clear exactly and clearing prices will be unique. Since nobody will have to be rationed, it is irrelevant which tie-breaking rule is used.

Payoffs.—In order to determine the optimal strategy, each agent maximizes his total surplus. This is defined as the total utility the agent receives from the goods minus his total payment. Following Kyle (1989), all initial endowments are normalized to 0. Owning quantities $\vec{q} \equiv (q_1 \ q_2)'$ an agent with type $\vec{s}_i \equiv (s_{i,1} \ s_{i,2})'$ receives a utility of

$$(2) \ U(q_1, q_2, \vec{s}_i) = \vec{s}_i' \vec{q} - \frac{1}{2} \vec{q}' \Delta \vec{q} \quad \text{where } \Delta \equiv \begin{pmatrix} \lambda & \delta \\ \delta & \lambda \end{pmatrix}, \lambda > 0, \delta \neq 0, |\delta| < \lambda.$$

This utility function is simple and intuitive: from winning \bar{q} , the agent obtains a marginal value \bar{s}_i . Holding an “inventory” \bar{q} of the illiquid assets is costly for the trader.¹⁰ He pays a marginal cost of Δ . It may be related to regulatory capital or collateral requirements of the assets or may represent an expected cost of being forced to raise liquidity by quickly disposing of remaining inventory in an illiquid market (Duffie and Zhu 2017). It can also come from substitutability or complementarities between the two securities, which are measured by parameter δ . When $\delta > 0$, the goods are substitutes, because the agent is willing to pay less for any additional amount of good m the more he purchases of “the other good,” which is indexed by

$$-m \equiv \{1,2\} \setminus m \text{ throughout the paper.}$$

Formally, $\partial U(q_1, q_2, \bar{s}_i) / \partial q_m = s_{i,m} - \lambda q_m - \delta q_{-m}$ decreases in q_{-m} . When $\delta < 0$, the goods are complements. By setting $\delta = 0$, I could shut down any interconnection between goods and revert to the case of an isolated auction. However, this case is uninteresting. With no relation between the goods, there are no strategic effects across goods. The allocation of the connected and disconnected market trivially coincides. I therefore focus on $\delta \neq 0$ throughout the paper. To guarantee that equilibrium demands are strictly decreasing in all possible scenarios, I let $|\delta| < \lambda$.

Equilibrium Concept.—The quadratic utility function renders the model tractable. In particular, it will allow me to derive closed-form solutions of the equilibrium functions. However, my optimality conditions for the Bayesian Nash equilibrium in simultaneous auctions hold for any utility function that is twice differentiable and has continuous cross-partial derivatives (see Lemma 8 in online Appendix F).

DEFINITION 2: *In the disconnected market, a pure-strategy BNE is a pair $\{b_{i,1}^*(\cdot, \bar{s}_i), b_{i,2}^*(\cdot, \bar{s}_i)\}$ that maximizes expected total surplus*

$$\max_{b_{i,1}(\cdot, \bar{s}_i), b_{i,2}(\cdot, \bar{s}_i)} E \left[U(\mathbf{q}_{i,1}^c, \mathbf{q}_{i,2}^c, \bar{s}_i) - \sum_m \mathbf{p}_m^c \mathbf{q}_{i,m}^c \right] \quad \forall i \in n,$$

where $\mathbf{q}_{i,m}^c = \mathbf{Q}_m - \sum_{j \neq i} x_{j,m}^*(\mathbf{p}_m^c, \bar{s}_j)$ is the amount agent i wins at the market-clearing price $\mathbf{p}_m^c = b_{i,m}(\mathbf{q}_{i,m}^c, \bar{s}_i)$ when all other players $j \neq i$ play the equilibrium strategies, $x_{j,m}^*(\cdot, \bar{s}_j) = b_{j,m}^{-1*}(\cdot, \bar{s}_j)$ for $m = 1, 2$.

¹⁰Equivalently, one may assume that agents are risk averse and share the same parameter of absolute risk aversion, normalized to 1. They differ in that they each expect different returns \mathbf{R}_i from two risky assets. An agent of type \bar{s}_i expects a CARA utility of $E[-\exp(-\bar{q}'\mathbf{R}_i)]$ from owing amounts \bar{q} of the securities, which generate jointly normally distributed payoffs, $\mathbf{R}_i \sim N(\bar{s}_i, \Delta)$. This is an alternative assumption on preferences, because the expected CARA utility is a monotone transformation of $U(\bar{q}, \bar{s}_i)$: $E[-\exp(-\bar{q}'\mathbf{R}_i)] = -\exp(-U(\bar{q}, \bar{s}_i))$ with $U(\bar{q}, \bar{s}_i) = \bar{s}_i' \bar{q} - (1/2) \bar{q}' \Delta \bar{q}$.

Related literature that only considers a single asset frequently assumes a quadratic cost $(1/2)\lambda q_m^2$; see Vives (2011), Rostek and Wernetka (2012), Duffie and Zhu (2017), and others.

The definition for the connected market is analogous, with the difference that both functions now depend on both quantities.

Since the true marginal willingness to pay, $\partial U(\bar{s}_i, q_1, q_2)/\partial q_m$, is linear and strictly decreasing in q_m ($\lambda > 0$), it is natural to look for BNE that are linear in all arguments and strictly decreasing in quantity/ies. This is common in the related literature.¹¹ Further, I focus on symmetric BNE in which all agents place bids for both goods given that all agents are ex ante symmetric and derive utility for both goods sold under ex ante symmetric market conditions.

DEFINITION 3: *A symmetric linear equilibrium is a pure-strategy BNE which is symmetric across agents and consists of strictly decreasing bidding functions that are linear in quantity/ies and types.*

II. Equilibrium

The irrelevance theorems build on a comparison of how much agents trade and pay in the symmetric linear equilibrium in a connected versus disconnected market. This equilibrium builds the basis for the formal analysis of the paper. In this section I demonstrate equilibrium existence and uniqueness. The remainder of the paper will provide detailed explanations of and intuition for how agents trade in this equilibrium. The corresponding bidding functions will be given in Propositions 2, 3, and 4.

PROPOSITION 1 (Equilibrium Existence and Uniqueness):

(i) *Connected market: There is a unique symmetric linear equilibrium.*

(ii) *Disconnected market:*

(a) *For any parameters $\{n, \lambda, \delta, \mu_{s_1}, \mu_{s_2}, \sigma_s, \rho_s, \mu_{Q_1}, \mu_{Q_2}, \sigma_Q, \rho_Q\}$ for which at least one real root ρ^* of the following polynomial*

$$(P) P(\rho) \equiv (\rho - \rho_Q)\sigma_Q^2(n-1)(\lambda n - \delta(n - 2(1 + \rho)))^2 \\ \times (\lambda n + \delta(n - 2(1 - \rho)))^2 \\ - \sigma_s^2(n-2)^2 n^2 \left(\delta^2 \left[\rho_s \left((n-2)^2 + 4(n-1)\rho^2 \right) - \rho(n^2 - 4 + 4\rho^2) \right] \right. \\ \left. + 2\delta\lambda n(2 - 2\rho^2 - n(1 - \rho\rho_s)) + \lambda^2 n^2(\rho_s - \rho) \right)$$

¹¹“Linear equilibria are tractable, particularly in the presence of private information, they have desirable properties like simplicity, and have proved to be very useful as a basis for empirical analysis” (Vives 2011, 1920). Studying them is the standard in related theoretic literature (e.g., Kyle 1989, Vives 2011, Rostek and Weretka 2012, Du and Zhu 2012, and Malamud and Rostek 2017). Some of these papers even constrain the strategy space to linear functions. My theoretic analysis is much broader in this regard, because I allow functions to be nonlinear in the interior of their domain. Support for linearity comes from the empirical literature on single-sided multiunit auction by Hortaçsu (2002). Using data from Turkish Treasury auctions, he shows that linear demands fit actual bidding behavior quite closely.

lies in $[-1, 1]$, there is a symmetric linear equilibrium. This equilibrium is unique if there is exactly one such ρ^* . Otherwise there is one equilibrium per ρ^* .

(b) For $\rho_s = \rho_Q = \pm 1$, there is a unique symmetric linear equilibrium.

The first part (i) of this proposition is an extension of the literature along the lines of Vives (2011), Rostek and Weretka (2012), and others, which have derived symmetric linear equilibria in uniform price auctions with a single good. I generalize their methods to an auction with multiple goods.

The second part (ii) does not rely on any existing work. Its first statement (a) applies to any possible correlation of types and total supply across goods. It provides sufficient conditions for equilibrium existence. A key ingredient of this equilibrium will be the correlation of market-clearing prices ρ^* . Given equilibrium play, ρ^* is implied by the exogenous correlations of types and supply. Characterized by $P(\rho^*) = 0$, this endogenous correlation is an equilibrium object itself. According to the fundamental theorem of algebra, the quintic polynomial $P(\cdot)$ always has at least one root in the space of real numbers. However, it must not necessarily lie in the relevant range of correlation coefficients: $[-1, 1]$. This condition becomes an implicit restriction on the exogenous parameters $\{n, \lambda, \delta, \mu_{s_1}, \mu_{s_2}, \sigma_s, \rho_s, \mu_{Q_1}, \mu_{Q_2}, \sigma_Q, \rho_Q\}$. The condition is always satisfied when types and total supply are perfectly correlated across markets. Then there always is a unique symmetric linear equilibrium (statement (b)).

The remainder of the section sketches the proof of Proposition 1. A reader who is less interested in technical details may jump to Section III, which includes an intuitive description of how agents choose their bidding functions.

SKETCH OF PROOF:

I derive the unique symmetric linear equilibrium in a guess-and-verify approach. Here I begin with the connected market. Denote $\vec{p} = (p_1 \ p_2)'$, $\vec{s}_i = (s_{i,1} \ s_{i,2})'$, guess that there is a symmetric linear equilibrium, and take the perspective of agent i of type \vec{s}_i . Let all other agents play the equilibrium guess. This implies that agent i trades against a residual supply curve that has the following form:

$$\vec{RS}(\vec{p}) = \vec{Q} - \sum_{j \neq i} \vec{x}(\vec{p}, \vec{s}_j) = \vec{Z} + f(\vec{p}),$$

with strictly increasing function $f(\cdot)$ and auxiliary random variable \vec{Z} . For fixed strategies of the other players, \vec{Z} aggregates all random factors (the supply, if random, and the types of all other agents). Importantly, the residual supply curve only varies in its intercept \vec{Z} . Therefore, the agent knows—for any prices \vec{p} that he might bid—the amount that he wins at market clearing: $\vec{q}^c = \vec{RS}_i(\vec{p})$. His best reply can be obtained by standard pointwise maximization:

$$(3) \quad \max_{\vec{p}} \{U(\vec{q}^c, \vec{s}_i) - \vec{p}'\vec{q}^c\} \quad \text{with} \quad \vec{q}^c = \vec{RS}_i(\vec{p}).$$

In determining the best reply in a disconnected market, the agent cannot condition his bids on the amounts that he might win of both goods, \bar{q}^c , because he is not allowed to submit such bids. This implies that the agent's best reply cannot be determined by pointwise maximization. Instead, one must maximize over all functions that the agent is allowed to submit (twice continuously differentiable functions on \mathbb{R} that are weakly decreasing and asymptotically linear). Denoting the set of such functions by \mathcal{B} , the maximization problem reads as follows:

$$(4) \quad \max_{p_1(\cdot) \in \mathcal{B}, p_2(\cdot) \in \mathcal{B}} E \left[U(\mathbf{q}_1^c, \mathbf{q}_2^c, \bar{s}_i) - \sum_{m=1,2} p_m(\mathbf{Z}_m) \mathbf{q}_m^c \right] \text{ with } \mathbf{q}_m^c = RS_m(p_m(\mathbf{Z}_m), \mathbf{Z}_m).$$

The expectation is taken over the intercepts of the residual supply curves, $\mathbf{Z}_1, \mathbf{Z}_2$. Denote their density function by $\phi(\cdot, \cdot)$, then

$$\max_{p_1(\cdot) \in \mathcal{B}, p_2(\cdot) \in \mathcal{B}} \int_{\mathbb{R}} \int_{\mathbb{R}} F(Z_1, Z_2, p_1(Z_1), p_2(Z_2)) \phi(Z_1, Z_2) dZ_1 dZ_2,$$

with $F(Z_1, Z_2, p_1(Z_1), p_2(Z_2)) = U(RS_1, RS_2, \bar{s}_i) - \sum_m p_m(Z_m) RS_m$, where I abbreviate $RS_m = RS_m(p_m(Z_m), Z_m)$ for $m = 1, 2$. To solve this problem, I proceed in several steps. I first assume that there is a pair $\{p_1^*(\cdot), p_2^*(\cdot)\}$ that solves (4). I derive necessary optimality conditions by computing the variational derivative (the analogue of the derivative when maximizing over variables). This involves constructing a class of comparison functions that vary $\{p_1^*(\cdot), p_2^*(\cdot)\}$ in different directions. I then show that a set of functions that fulfills the optimality conditions is a maximum. Generally, it may be very challenging to obtain sufficient conditions when maximizing over functions. My problem becomes tractable because I can show that $F(Z_1, Z_2, \cdot, \cdot)$ is for any Z_1, Z_2 strictly concave as a function of p_1, p_2 . The next step is to prove existence of $\{p_1^*(\cdot), p_2^*(\cdot)\}$ by pointwise solving the set of optimality conditions. The solution is a pair of linear functions when all random variables are normally distributed. Each of these functions specifies a price per Z_m . The bidding functions I seek to characterize instead specify a price for every quantity point q_m . I can recover them from $\{p_1^*(\cdot), p_2^*(\cdot)\}$ since there is, for every q_m on any bidding function that the agent may submit, a unique realization Z_m . This is because the residual supply curves are strictly increasing and bidding functions must be decreasing.

Having characterized the agent's best reply, I obtain the symmetric equilibrium for both market structures by matching the (unique) coefficients of the agent's best reply with the equilibrium guess that is played by all other agents. ■

III. Irrelevance Theorems

At the start of any economic analysis, it is useful to build an initial intuition. In the connected market, traders with joint preferences over the goods for sale are allowed to bid for bundles and can thereby jointly maximize their total surplus. Instead, in a disconnected market, their demand schedule can only depend on the price of the security traded in that market. By design of the transaction, agents are *always* constrained regarding how they can display their preferences. One would therefore

expect that the equilibrium allocation of the connected market must differ from that of the disconnected market. My irrelevance theorems prove that this intuition can be misleading.

Section IIIA analyzes “small” markets with finitely many strategic agents ($n < \infty$). The first theorem establishes *necessary* and *sufficient* conditions under which connected and disconnected markets give rise to identical market outcomes. The second theorem generalizes the results for markets that clear at imperfectly correlated prices and demonstrates that market outcomes coincide in expectation. Section IIIB focuses on “large” markets with infinitely many agents ($n \rightarrow \infty$). In the limit when all traders become price-takers, both clearing prices and quantities always coincide ex post. Section IIIC uses data from the primary market of government bonds to illustrate that these equivalence results can help us understand why some markets remain disconnected even though they could be connected.

A. Small Markets

When $n < \infty$, each agent faces a nonzero probability of affecting the market-clearing price with his price offers. Therefore, all agents have incentives to behave strategically and misreport their true willingness to pay.

Perfect Correlation.—The first irrelevance theorem tells us that both market structures give rise to identical outcomes if and only if the underlying uncertainties about the goods (the private types and the total supply) are perfectly correlated.

IRRELEVANCE THEOREM 1: *Let $n < \infty$.*

- (i) *The equilibrium allocation $\{q_{i,1}^*, q_{i,2}^*\}_{i=1}^n$ of symmetric linear equilibria in connected and disconnected markets always coincide ex post if and only if*

$$\left((\rho_s = \rho_Q = \pm 1) \text{ or } (\rho_s = \pm 1 \text{ and } \sigma_Q = 0) \right) \Leftrightarrow \rho^* = \pm 1.$$

- (ii) *The associated clearing prices $\{p_1^*, p_2^*\}$ differ if and only if $\mu_{Q_1} \neq \rho_Q \mu_{Q_2}$.*

- (iii) *Both statements hold in the limit as $\rho^* \rightarrow \pm 1$.*

Before deriving an intuition for why and when quantities and prices coincide, I would like to point out that perfect correlation does not imply that the two disconnected markets are identical copies of one another. While both market environments are symmetric from an ex ante perspective, given that all distributions are symmetric, they typically differ ex post. This can also be the case under perfect correlation. To illustrate this point, consider a numeric example. For $n = 30$ agents, I draw from the distribution of types and supply with $\rho_s = \rho_Q = -1$, $\mu_{s_1} = 10$, $\mu_{s_2} = 11$, and $\sigma_s = 1$, $\sigma_Q = 10$. Noise traders provide $Q_1 \approx 3.5$ and $Q_2 \approx -3.5$. Figure 1 displays the realized types (y -axis) for each agent (x -axis). Both exogenous supply and the agents’ types—and with them their marginal willingness to pay for the goods—differ across markets.

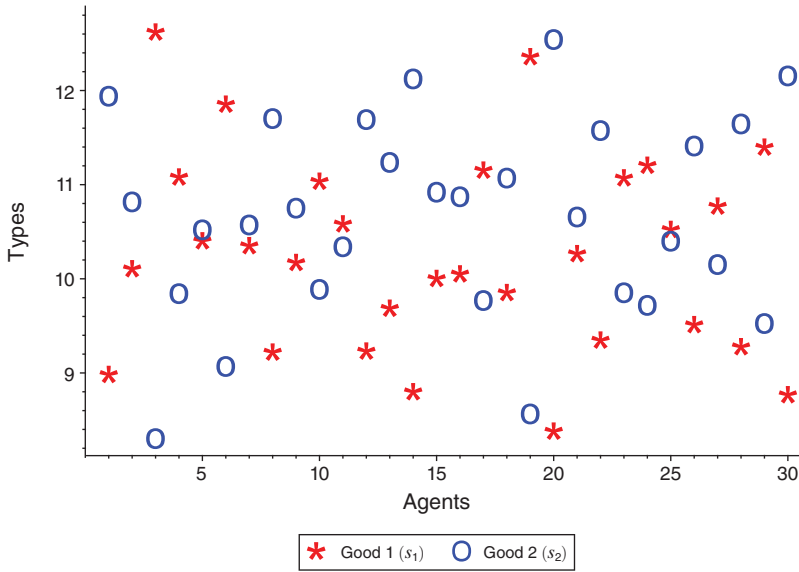


FIGURE 1. EXAMPLE OF TYPE PROFILE

(i) **Why Quantities Coincide.**—To provide an intuition for the first statement of the theorem, I describe how agents choose their equilibrium strategies, i.e., solve maximization problems (3) and (4). In the connected market, the agent goes through all possible pairs of realizations of residual supply curves. Say that a particular pair realizes and that offering prices $\{p_1, p_2\}$ makes agent i win $\{q_1, q_2\}$. For this bid to be optimal, it must clear the market and equate the marginal utility from winning the bid with the required additional payment.¹² This is not feasible when the market is disconnected, because the agent does not know how much he will win and pay in the other auction. To determine the optimal bids for good 1, the agent goes through all possible realizations of the residual supply curve in auction 1. For each amount q_1 that he might win, he makes the best guess about the outcome of the other auction: he forms an expectation over the clearing price p_2^* and the amount that he wins $q_{i,2}^*$, conditional on winning q_1 of good 1. An optimal bid then equates the *expected* marginal benefit with the *expected* marginal payment and clears the market.¹³

This implies that the agent can generally make a relatively better informed decision in the connected market. There is an exception: If and only if the residual supply curves (and with them the clearing prices) are perfectly correlated—which is the case when the marginal types $(s_{i,1}, s_{i,2})$ and (if random) the supplies (Q_1, Q_2) are perfectly correlated across goods—then the agent knows exactly how much he wins and pays in the other auction in the event of winning a particular amount q_1 . This is because a realization of the residual supply curve of good 1 maps one-to-one to some realization of the curve of good 2. Facing the same degree of uncertainty, the agent wins the same amounts under both market structures given that markets clear under the same auction rule.

¹²Lemma 5 in online Appendix D4 formally summarizes this optimality condition.

¹³Lemma 6 in online Appendix D4 formally summarizes this optimality condition.

The intuition that drives this result should generalize to many other environments that are not considered on a formal level. Say there are some underlying uncertainties about good 1. Other than from private information or noise traders, those uncertainties could come from affiliated or common values for the assets, idiosyncratic inventory shocks, and much more. The key is that for given strategies of the other agents, these underlying uncertainties aggregate to some random variable $\mathbf{Z}_{i,1}$ that governs the residual supply for good m : $RS_{i,1}(p_1, \mathbf{Z}_{i,1})$. In this type of setting, I expect the equilibrium allocation of the disconnected and connected markets to coincide not only when $\mathbf{Z}_{i,1}$ and $\mathbf{Z}_{i,2}$ are perfectly correlated but, more generally, when they move one-to-one.¹⁴

(ii) Why Prices Differ.—The difference in clearing prices comes from a difference in bidding behavior that arises even when types and total supply are perfectly correlated across markets, $\rho^* = \pm 1$. By the rules of the auction, the clearing price equals the bid that agents make for the amount they win in equilibrium: $\bar{p}_m^* = \bar{b}_m(\bar{q}_{i,m}^*, \bar{q}_{i,-m}^*, \bar{s}_i)$ and $p_m^* = b_m(q_{i,m}^*, \bar{s}_i)$ for $m = 1, 2$. To understand what drives price differences, it is useful to compare the bidding functions that agents submit across market structures.

PROPOSITION 2: *In the unique symmetric linear equilibrium of the connected market, traders submit*

$$(5) \quad \bar{b}_m^*(q_m, q_{-m}, s_{i,m}) = s_{i,m} - \left(\frac{n-1}{n-2}\right)(\lambda q_m + \delta q_{-m}) \quad \text{for } m = 1, 2.$$

In equilibrium, each agent shades his true marginal willingness to pay, $\partial U(q_1, q_2, \bar{s}_i) / \partial q_m = s_{i,m} - \lambda q_m - \delta q_{-m}$, as in Ausubel et al. (2014). Such strategic bid shading also plays a role in the disconnected market, in which the agent’s price offer for good m cannot depend on the amount that he would like to trade of good $-m$. The agent is forced to substitute q_{-m} in $\bar{b}_m^*(q_m, q_{-m}, \bar{s}_i)$ by some function of q_m . He can no longer interlink his submitted demands *explicitly*. When the residual supply curves are perfectly correlated, however, he can interlink his demands *implicitly*. The following proposition shows how.¹⁵

¹⁴This is a weaker condition than perfect correlation, as it allows the realizations of both variables to be interlinked by some deterministic function $f(\cdot)$ that is not necessarily linear: $\mathbf{Z}_{i,2} = f(\mathbf{Z}_{i,1})$. Perfect correlation instead defines a linear relation: $\mathbf{Z}_{i,2} = l(\mathbf{Z}_{i,1}) \equiv E[\mathbf{Z}_{i,2}] - gE[\mathbf{Z}_{i,1}] + g\mathbf{Z}_{i,1}$ with $g = \pm\sqrt{\text{var}(\mathbf{Z}_{i,2})/\text{var}(\mathbf{Z}_{i,1})}$. Without it, equilibria will no longer be linear. While it is mathematically much more challenging to solve for nonlinear equilibria, my intuition does not rely on linearity. Instead, it is based on analyzing the agent’s best reply defined by his optimality conditions. While I have focused the discussion on linear equilibria, I show in online Appendix F that the bidding incentives in disconnected markets are driven in an analogous fashion when equilibria are not necessarily linear—for instance, because the utility function is not linear (Lemma 8). Therefore, I do not expect this result to hinge on the linearity of strategies.

¹⁵The equilibrium of Proposition 3 may exist even when types and supply are not normally distributed but follow other distributions. This is interesting, because the underlying optimality condition hinges (with quadratic utility) on a conditional expectation $E[\mathbf{q}_{i,-m}^* | q_m]$, which is typically not linear. The solution is independent of any particular distribution, because i ’s winning quantity in market $-m$ is a linear function $l(\cdot)$ of i ’s winning quantity in market m when clearing prices are perfectly correlated: $\mathbf{q}_{i,-m}^* = l(\mathbf{q}_{i,m}^*)$. As a result, both conditional expectations are linear in quantity.

PROPOSITION 3: *Let $\rho^* = \pm 1$. In the unique symmetric linear equilibrium of the disconnected market, traders submit*

$$(6) \quad b_m^*(q_m, s_{i,m}) = \bar{b}_m^*(q_m, \rho^* q_m, s_{i,m}) + \delta \left(\frac{1}{n} \right) (\rho^* \mu_{Q_m} - \mu_{Q_{-m}}) \quad \text{for } m = 1, 2.$$

An agent trading in a disconnected market asks for a mark-up (or discount) of $\delta(1/n)(\rho^* \mu_{Q_m} - \mu_{Q_{-m}})$ relative to the price he offers for good m in a connected market when winning a bundle of amounts $\{q_m, \rho^* q_m\}$. Intuitively, the price mark-up reflects information about (noise) trading that an agent has to estimate when the market is disconnected but which can be fully incorporated in the prices when the market is connected.

This is most easily understood when abstracting from the strategic aspect of the bidding process. Assume for the moment that agents are truthful. In the connected market, each agent submits his true marginal willingness to pay. For good 1: $\bar{b}_1(q_1, q_2, s_{i,1}) = s_{i,1} - \lambda q_1 - \delta q_2$. In the disconnected market, the agent must take an expectation over how much he will win in the other market, $\mathbf{q}_{i,2}^*$. As I explain in Section IIIB, he submits $b_1(q_1, s_{i,1}) = s_{i,1} - \lambda q_1 - \delta E[\mathbf{q}_{i,2}^* | q_1]$. Under perfect correlation and with ex ante identical agents, the conditional expectation is a linear function of q_1 , namely $E[\mathbf{q}_{i,2}^* | q_1] = (\mu_{Q_2}/n) - \rho^*((\mu_{Q_1}/n) - q_1)$. Inserting this term into $b_1(q_1, s_{i,1})$ and comparing it to $\bar{b}_1(q_1, q_2, s_{i,1})$, one finds that (6) also holds when agents are truthful.

The mapping between bids translates into a mapping between clearing prices, which solve $np_1^* = \sum_{i=1}^n b_1(q_{i,1}^*, s_{i,1})$ and $n\bar{p}_1^* = \sum_{i=1}^n \bar{b}_1(\bar{q}_{i,1}^*, \bar{q}_{i,2}^*, s_{i,1})$. When agents are truthful and correlation is perfect, the clearing prices in the disconnected and the connected market coincide.¹⁶ When agents are strategic, the clearing prices differ because of differences in the bid shading. To see this, compute the clearing prices by aggregating bidding functions (5) and (6) and take differences: $p_1^* - \bar{p}_1^* = (\delta/n)(\rho^* \mu_{Q_1} - \mu_{Q_2})(1 - (n-1)/(n-2))$. Relative to the case in which agents are truthful, there is an additional term: $1 - (n-1)/(n-2) \neq 0$. It comes from differential bid shading in the connected market, in which the agent decreases his bid for good 1 by $\delta((n-1)/(n-2))$ rather than δ when q_2 increases by 1 unit. The following corollary summarizes the mapping between clearing prices.

COROLLARY 1: *Let $\rho^* = \pm 1$. A linear mapping relates the realized clearing prices of the two market structures:*

$$(7) \quad p_m^* = \bar{p}_m^* - \left(\frac{\delta}{n} \right) \left(\frac{1}{n-2} \right) (\rho^* \mu_{Q_m} - \mu_{Q_{-m}}) \quad \text{for } m = 1, 2.$$

This relationship implies that market-clearing prices can be ranked. For example, when $\rho^* = 1$ and $\mu_{Q_1} > \mu_{Q_2}$; $p_1^* < \bar{p}_1^*$ and $\bar{p}_2^* < p_2^*$ for substitutes ($\delta > 0$) and vice versa for complements ($\delta < 0$).

¹⁶For good 1, for example, $p_1^* = (1/n) \sum_{i=1}^n s_{i,1} - (1/n)(\lambda + \delta \rho^*) Q_1 + (\delta/n)(\rho^* \mu_{Q_1} - \mu_{Q_2})$ is equivalent to $\bar{p}_1^* = (1/n) \sum_{i=1}^n s_{i,1} - (1/n)(\lambda Q_1 + \delta Q_2)$ because $Q_2 = \mu_{Q_2} - \rho^*(\mu_{Q_1} - Q_1)$ when $|\rho^*| = 1$.

Imperfect Correlation.—What happens when the clearing prices are imperfectly correlated? From the irrelevance theorem, we already know that traded quantities and clearing prices differ ex post across market structures. From the ex ante perspective—that is, before agents draw their types and supply realizes—they might coincide. This section shows that equivalence is preserved in expectation. To facilitate the comparison with the previous section, I keep the same structure of the text. In contrast to the results of the previous section, it should be noted that the following results rely on the linearity property of the expectation operator and therefore might not generalize to settings with nonlinear equilibria.

IRRELEVANCE THEOREM 2: *Let $n < \infty$.*

(i) *The expected equilibrium allocations $E[\mathbf{q}_{i,1}^*], E[\mathbf{q}_{i,2}^*]$ of symmetric linear equilibria in connected and disconnected markets always coincide.*

(ii) *The associated expected clearing prices $E[\mathbf{p}_1^*], E[\mathbf{p}_2^*]$ coincide if and only if $\mu_{Q_1} = \rho_Q \mu_{Q_2}$.*

(i) Why Quantities Coincide.—It is not surprising that ex ante identical agents expect to pay and win the same amounts *within* the same market structure. What is not obvious is equivalence *across* market structures. How much an agent ends up winning in both the disconnected and connected market depends on the realized total supply $\{Q_1, Q_2\}$, his type, and those of all others $\{s_{i,1}, s_{i,2}\}_{i=1}^n$. What matters in particular is how much the agent values the good *relative* to the average valuation in the market. Who draws a type $s_{i,m}$ above the average of all market participants $(1/n) \sum_i s_{i,m}$ ends up winning more of good m than one that lies below it. From an ex ante perspective, all agents expect to value the good like all others: $E[s_{i,m}] = \mu_{s_m}$. As a result, all expect to win an equal share of the total supply: $E[\mathbf{q}_{i,m}^*] = E[\bar{\mathbf{q}}_{i,m}^*] = \mu_{Q_m}/n$.

(ii) Why Prices Differ.—As above, price differences arise from differences in bidding behavior. From Proposition 2 (which holds for any correlation), we already know the bidding function of the connected market. The following proposition states the equilibrium bidding functions in the disconnected markets for any correlation, $\rho^* \in [0, 1]$.¹⁷

PROPOSITION 4: *In any symmetric linear equilibrium of the disconnected market, traders submit for $m = 1, 2$,*

$$(8) \quad b_m^*(q_m, \vec{s}_i) = (\lambda + \delta\rho^*) \left[\frac{(\lambda n + 2\delta\rho^*)ns_{i,m} - \delta(\lambda + \delta\rho^*)(n - 2)ns_{i,-m}}{[\lambda n - \delta(n - 2(1 + \rho^*))][\lambda n + \delta(n - 2(1 - \rho^*))]} - \left(\frac{n-1}{n-2}\right)q_m \right] + C_m,$$

¹⁷The equilibrium analysis becomes more complex with imperfect correlations. In particular, we lose the linear mapping between the equilibrium winning quantities of both auctions without an appropriate distributional (here Gaussian) assumption, which ensures that the conditional expectation of the winning quantity of the other auction is a linear function. Only then can there be linear equilibria.

with correlation ρ^* implicitly defined as the real root of $P(\rho)$ of Proposition 1 and $C_m = (\delta/n)[\rho^* \mu_{Q_m} - \mu_{Q_{-m}}] - \left(\frac{n-2}{4}\right) \times \left[2\mu_{s_m} + \frac{n(\delta-\lambda)(\mu_{s_m} - \mu_{s_{-m}})}{\lambda n - \delta\{n-2(1+\rho^*)\}} - \frac{n(\delta+\lambda)(\mu_{s_m} + \mu_{s_{-m}})}{\lambda n + \delta(n-2(1-\rho^*))}\right]$.

In a nutshell, strategic incentives lie in between two extreme cases: When correlation is perfect, the agent behaves *as if* both goods were traded in one connected market, with the difference that he can only interlink his demand implicitly—and not explicitly—across goods (Proposition 3). With zero correlation between auctions, the agent cannot use any information in one market to better predict the outcome of the other. No matter how much he demands in one market, it will not affect the market clearing of the other. Essentially, the agent behaves *as if* he were competing in an isolated market with no outside option. He only discounts his true marginal valuation, $s_{i,m}$, by how much he expects to win of the related good offered in the parallel auction, $\delta E[\mathbf{q}_{i,-m}^*]$.¹⁸

When taking expectations, the complicated bidding function (8) simplifies. Specifically, the prices that agents *expect* to pay display the same relation across market structures as the prices they actually pay *ex post* under perfect correlation (see Proposition 3).

COROLLARY 2: *Ex ante, traders expect to pay*

$$(6') \quad E[b_m^*(\mathbf{q}_{i,m}^*, \bar{\mathbf{s}}_i)] = E[\bar{b}_m^*(\mathbf{q}_{i,m}^*, \rho^* \mathbf{q}_{i,m}^*, \bar{\mathbf{s}}_i)] + \delta \left(\frac{1}{n}\right) (\rho^* \mu_{Q_m} - \mu_{Q_{-m}})$$

for $m = 1, 2$ in any symmetric linear equilibrium of the disconnected market.

To win amount $E[\mathbf{q}_{i,m}^*] = \mu_{Q_m}/n$ in disconnected market m , the agent expects to pay the mark-up that we know from Proposition 3 relative to the price he expects to pay for good m when winning a bundle of amounts $\{E[\mathbf{q}_{i,m}^*], E[\rho^* \mathbf{q}_{i,m}^*]\} = \{\mu_{Q_m}/n, \rho^*(\mu_{Q_m}/n)\}$ in the connected market. The intuition for this linkage is analogous to the one for Proposition 3: The agent anticipates that he has to make an educated guess about how much he wins in the other auction when the market is disconnected, but not when it is connected. This, in turn, affects how much he will be willing to pay (and with it the aggressiveness of bidding) differently across market structures. Analogous to Corollary 2, this relationship implies the following mapping between expected clearing price.

¹⁸Formally, when $\rho^* = 0$, $b_m^*(q_m, s_{i,m}) = b_m^{IA}(q_m, s_{i,m}^{IA})$ with $s_{i,m}^{IA} \equiv s_{i,m} - \delta E[\mathbf{q}_{i,-m}^*]$, where $b_m^{IA}(\cdot, s_{i,m}^{IA})$ denotes the bidding function of an agent with type $s_{i,m}^{IA}$ in a standard single-good uniform-price double auction that takes place in isolation.

COROLLARY 3: *A linear mapping relates the expected clearing prices of the two market structures:*

$$(7') \quad E[\mathbf{p}_m^*] = E[\bar{\mathbf{p}}_m^*] - \left(\frac{\delta}{n}\right) \left(\frac{1}{n-2}\right) (\rho^* \mu_{Q_m} - \mu_{Q_{-m}}).$$

The corollary implies that the expected prices differ more strongly across market structures the higher the expected asymmetry in the exogenous supply: $|\rho^* \mu_{Q_m} - \mu_{Q_{-m}}|$. Intuitively, one would expect the same to hold as the dependence between goods increases, $|\delta| \uparrow$, and the fewer strategic agents trade, $n \downarrow$. Formally, comparative statics are difficult to derive because ρ^* depends nontrivially on δ, n .

B. Large Markets

When markets grow large, all individuals lose their market power. In the limit, as $n \rightarrow \infty$, there are so many agents that no one affects the market-clearing price with positive probability.

IRRELEVANCE THEOREM 3: *As $n \rightarrow \infty$, the equilibrium allocation $\{q_{i,1}^*, q_{i,2}^*\}_{i=1}^n$ and clearing prices $\{p_1^*, p_2^*\}$ of symmetric linear equilibria in connected and disconnected markets converge toward one another.*

Equilibrium allocations and the prices of connected and disconnected markets with price-taking agents always coincide. The reason is twofold. First, uncertainty about exogenous supply by noise traders washes out, because so many “informed” agents trade. Second, price-taking agents are truthful. To see this, consider Corollary 4. It displays bidding behavior in markets with finitely many agents.

COROLLARY 4: *Competing with infinitely many agents, traders choose*

$$\bar{b}_m^*(q_m, q_{-m}, \bar{s}_i) = \partial U(q_m, q_{-m}, \bar{s}_i) / \partial q_m \quad \text{in the connected market and}$$

$$b_m^*(q_m, \bar{s}_i) = \lim_{n \rightarrow \infty} E[\partial U(q_m, \mathbf{q}_{i,-m}^*, \bar{s}_i) / \partial q_m | q_m] \quad \text{in the disconnected market}$$

for $m = 1, 2$, in any linear BNE.

In connected markets, in which price-taking agents are free to submit their true marginal willingness to pay, being truthful just means reporting those. In disconnected markets, this is not possible by the rules of the transactions. Here, Feldman et al. (2016) correctly points out that agents do not submit their true [combinatorial] valuations. This does not imply, however, that they are not truthful. When evaluating whether agents are truthful, one must consider the constraints given by the rules of transaction. What determines whether an agent is truthful in *parallel* uniform-price auctions is not how much the agent *actually* benefits marginally from purchasing a bit of good m but what he *expects* to receive. With this in mind, one recognizes that price-taking agents of disconnected markets do not ask for any mark-up above their

truthful (expected) valuations. Since agents do report their preferences truthfully and either market clears under the same uniform pricing rule, both market structures achieve the same outcome in the limit when $n \rightarrow \infty$.

C. Empirical Illustration

I conclude the discussion with an example of a market that is disconnected even though it could be connected relatively easily. It demonstrates that my theoretic results may be valuable for comparing markets using aggregate data on prices/yields. Such data are, in contrast to transaction-level data, publicly available for many financial markets.

The example comes from the primary market of government bonds. Several countries, including the world's largest economies (the United States, China, Japan, Germany, France, India, Brazil, and Canada), issue bonds of different maturities in parallel yet separate multiunit auctions. Many of them, including the United States, run uniform-price auctions. Since bidders only buy and do not trade, the model must be modified slightly to fit this application, in that demand functions cannot drop below zero. The other key features of my models fit the example well: the most influential bidders (the primary dealers) participate in all auctions, (conditionally independent) private valuations seem to drive their bidding behavior (as tested by Hortaçsu and Kastl 2012), and random supply comes either from the auctioneer who adapts the total amount for sale during the auction or from noncompetitive tenders who demand an amount without specifying a price and are won with certainty.

I compute the correlation of the clearing prices of US Treasury bill auctions using data from Treasury Direct (2019) (see Table 1). The data sample runs from November 2, 1998, until September 18, 2018, and includes the four maturities of US Treasury bills: 4, 13, 26, and 52 weeks. It counts 1,925 auction dates. The 13- and 26-week bills were mostly issued in parallel auctions. The 4- and 52-week bills were also issued on separate yet closely following dates.¹⁹ Not surprisingly, the correlation coefficient is highest (0.9995) for bills that are typically auctioned on the same day, i.e., the 13- and 26-week bills.

My equivalence results (Theorem 1) suggest an explanation for why debt management offices might not bother to connect this disconnected market. They could, for example, implement a combinatorial auction such as Klemperer's (2010) product-mix auction, in which bidders can buy bundles of the three bills. According to my result, such a change in the auction design would not cause large changes in the market outcome and might not be worthwhile.

¹⁹The current auction calendar is available at <https://www.Treasury.gov/resource-center/data-chart-center/quarterly-refunding/Documents/auctions.pdf>.

TABLE 1—CORRELATIONS IN US TREASURY AUCTIONS

	P_{4wk}^*	P_{13wk}^*	P_{26wk}^*	P_{52wk}^*
P_{4wk}^*	1.00			
P_{13wk}^*	0.9969	1.00		
P_{26wk}^*	0.9952	0.9995	1.00	
P_{52wk}^*	0.9922	0.9982	0.9971	1.00

Note: Price correlations in US T-bill auctions from 11/1998 to 09/2018.

Source: TreasuryDirect

IV. Welfare Implications

This section compares the market performance of connected and disconnected markets. Taken together, the results give us an idea about the benefits and costs of connecting disconnected markets by allowing for expressive bidding languages, and they illustrate how these vary with the correlation between the two goods and with the nature of the goods (complements versus substitutes).

Analogous to Section III, Section IVA first analyzes small markets whose prices are perfectly correlated. Section IVB then considers small markets under imperfect correlation, and Section IVC concludes with large markets.

A. Small Markets with Perfect Correlation

This section concentrates on small markets whose clearing prices are perfectly correlated: $\rho^* = \pm 1 \Leftrightarrow ((\rho_s = \rho_Q = \pm 1) \text{ or } (\rho_s = \pm 1 \text{ and } \sigma_Q = 0))$. It investigates which market structure is (i) more efficient and (ii) more attractive for strategic agents. I consider two measures of market performance: “total utility” and “total surplus.” The former is the aggregate utility of all strategic traders from winning the equilibrium quantities. The latter includes the payments they make to purchase or receive to sell those amounts.

DEFINITION 4: Consider a disconnected market. Define

$$U^* \equiv \sum_{i=1}^n U(q_{i,1}^*, q_{i,2}^*, \bar{s}_i), \tag{Total Utility}$$

$$TS^* \equiv U^* - \sum_{m=1}^2 TP_m^*, \tag{Total Surplus}$$

with

$$TP_m^* \equiv \sum_{i=1}^n p_m^* q_{i,m}^*$$

as total payments for m . Define \bar{U}^* , \bar{TS}^* , and \bar{TP}_m^* accordingly for the connected market.

Notice that in a world without noise traders ($Q_1 = Q_2 = 0$), the total surplus is the same as the total utility: $U^* = TS^*$. It is the aggregate utility that *all* agents in the economy achieve in equilibrium and is typically referred to as “welfare.” Since payments only represent transfers across all agents in the economy, they are neutral: $TP_m^* = 0$. With noise traders, the total payments of strategic agents are only one part of all transfers in the market. More explicitly $TP_m^* \neq 0$ is the total amount that strategic agents receive from or make to noise traders. Loosely speaking, it measures how much large, experienced traders can take advantage of less experienced or uninformed agents.

COROLLARY 5: *Let $n < \infty$ and $\rho^* = \pm 1$.*

- (i) *Both market structures achieve the same degree of efficiency: $U^* = \bar{U}^*$.*
- (ii) *The total surplus might differ:*

$$TS^* \begin{cases} = \bar{TS}^* & \text{for } \rho^* \mu_{Q_1} = \mu_{Q_2} \\ > \bar{TS}^* & \text{for } \rho^* \mu_{Q_1} \neq \mu_{Q_2} \text{ if and only if } (\rho^* = +1 \text{ and } \delta > 0) \\ & \text{or } (\rho^* = -1 \text{ and } \delta < 0) \\ < \bar{TS}^* & \text{for } \rho^* \mu_{Q_1} \neq \mu_{Q_2} \text{ if and only if } (\rho^* = -1 \text{ and } \delta > 0) \\ & \text{or } (\rho^* = +1 \text{ and } \delta < 0). \end{cases}$$

By statement (i) of Theorem 1, quantities coincide when correlation is perfect. It immediately follows that both market structures achieve the same total utility. They are equally efficient. Clearing prices may differ by statement (ii) of Theorem 1. In consequence, the total payments that strategic agents make (all together) differ and cause the total surplus to differ. To see this, consider the example of perfect positively correlated ($\rho^* = 1$) substitutes ($\delta > 0$). Both market structures give rise to the same total surplus only when strategic agents expect there to be the same size of exogenous supply for both goods ($\mu_{Q_1} = \mu_{Q_2}$). Otherwise the disconnected market outperforms the connected one from the perspective of the strategic agents, $TS^* > \bar{TS}^*$, because strategic agents pay a more favorable price in trades with noise traders by Corollary 1. An analogous argument holds for all other cases, which are summarized by the following corollary. It states the differences in total payments implied by the differences in clearing prices.

COROLLARY 6: *Let $n < \infty$, $\rho^* = \pm 1$, $\mu_{Q_1} \leq \rho^* \mu_{Q_2}$, and $\delta > 0$ (substitutes).*

- (i) $TP_1^* = \bar{TP}_1^*$ and $TP_2^* = \bar{TP}_2^*$ if $\mu_{Q_1} = \rho^* \mu_{Q_2}$, otherwise
- $TP_1^* > \bar{TP}_1^*$ and $TP_2^* < \bar{TP}_2^*$ if $\rho^* = +1$,
- $TP_1^* < \bar{TP}_1^*$ and $TP_2^* < \bar{TP}_2^*$ if $\rho^* = -1$.

$$\begin{aligned}
 \text{(ii)} \quad \sum_m TP_m^* &= \sum_m \overline{TP}_m^* \text{ if } \mu_{Q_1} = \rho^* \mu_{Q_2}, \text{ otherwise} \\
 \sum_m TP_m^* &< \sum_m \overline{TP}_m^* \text{ if } \rho^* = +1, \\
 \sum_m TP_m^* &> \sum_m \overline{TP}_m^* \text{ if } \rho^* = -1.
 \end{aligned}$$

The opposite inequalities hold for complements ($\delta < 0$).

In total, strategic agents pay strictly less for positively correlated substitute goods when the market is disconnected, unless $\mu_{Q_1} = \rho^* \mu_{Q_2}$. This finding provides one possible reason for why many financial markets are fragmented. It offers an alternative argument for why there are dozens of different venues that all trade equity and fixed income securities: strategic traders, such as dealers, mutual funds, and pension funds, may prefer the markets to be disconnected because they can trade with less experienced or less informed traders at a price that is more favorable (for themselves). If large financial institutions have lobbying power over how markets are organized, it is not surprising that there is no trend to integrate fragmented markets.

B. Small Markets with Imperfect Correlation

In the previous section, I focused on differences between strategic agents and noise traders if markets are perfectly correlated. In this section, I zoom in on strategic agents and allow correlation to be imperfect. With imperfect correlation, the equilibrium allocation no longer coincides ex post (Theorem 1), and it quickly becomes intractable to compute how much total surplus realizes, both on the aggregate and the individual level.²⁰

To obtain insights into who would win and who would lose were one to connect disconnected markets, I provide two stylized examples. They characterize which agents (i.e., which types) obtain a higher (individual) total surplus under the disconnected versus the connected market in specific market environments. I then analyze which market structure is more efficient, in that it achieves higher welfare. For this, I abstract from noise traders whose preferences are undefined to obtain a clean measure of welfare, $TS^* = U^*$ (as defined in Definition 4) by setting $\mu_{Q_1} = \mu_{Q_2} = \sigma_Q = 0$. Bringing back the noise traders, I conclude with a sufficient condition that tells us when strategic traders expect a higher total surplus in the disconnected market than the connected market.

Individual Surplus.—Both examples consider a perfectly symmetric market environment ($\mu_{s_1} = \mu_{s_2} = \mu_s$) without noise traders ($\mu_{Q_1} = \mu_{Q_2} = \sigma_Q$). To shut down differences between TS_i^* and \overline{TS}_i^* that might purely be driven by differences in prices rather than quantities, I set the realized average type for both goods equal to its true expected value, $\mu_{s_m} = (1/n) \sum_i s_{i,m}$ for $m = 1, 2$, so that both market structures clear at the same prices, namely $p_m^* = \overline{p}_m^* = \mu_s$.

²⁰Note that it is straightforward to compute aggregate welfare $W^* = \sum_i TS_i^*$ for specific numeric examples by inserting winning quantities and clearing prices of Lemma 4 in online Appendix B into U_i^* and TP_i^* and aggregating across individuals.

The first example considers an agent who draws the same type in both markets. Under imperfect correlation, such a draw will almost never realize. It is nevertheless useful to explain the basic intuition by means of this extreme case.

Example 1: Let $\rho^* \in (-1, 1)$, $\mu_{s_1} = \mu_{s_2} = (1/n) \sum_i s_{i,1} = (1/n) \sum_i s_{i,2} \equiv \mu_s$.

An agent who draws $s_{i,1} = s_{i,2} = s$ realizes a higher total surplus in the disconnected market than the connected market when goods are substitutes, and vice versa for complements:

$$TS_i^* - \overline{TS}_i^* = \delta \times g(n, \lambda, \delta, \rho^*) \times (\mu_s - s)^2 \begin{cases} \geq 0 & \text{for } \delta > 0 \\ \leq 0 & \text{for } \delta < 0 \end{cases}$$

with $g(n, \lambda, \delta, \rho^*) \equiv 4(n-2)n(1-\rho^*)(\lambda + \delta\rho^*) \left[(n-1)(\lambda + \delta \times (n-2(1-\rho^*))) \right]^{-2} > 0$.

Since markets clear at the same prices $p_m^* = \bar{p}_m^*$ by assumption and agents pay the clearing price for all units they win, only quantity differences matter. More specifically, the agent prefers the market to be disconnected when he wins an amount that is closer to the amount he would win if everyone in the market were truthful so that the allocation is not distorted.

To see this, compute the amount that a price-taking agent would choose. He maximizes $TS_i(s_i, q_1, q_2) = U(s_i, q_1, q_2) - \sum_m p_m^* q_m$ subject to $p_m^* = \mu_{s_m}$ by picking $q_{i,m}^e = a_1^e s_{i,m} + a_2^e s_{i,-m}$ with $a_1^e = \lambda(\lambda^2 - \delta^2)$ and $a_2^e = -\delta(\lambda^2 - \delta^2)$. Comparing this amount to the amount the agent actually wins in equilibrium—which is $q_{i,m}^* = a_1^* s_{i,m} + a_2^* s_{i,-m}$ in the disconnected market and $\bar{q}_{i,m}^* = \bar{a}_1^* s_{i,m} + \bar{a}_2^* s_{i,-m}$ in the connected market, where $\{a_1^*, a_2^*, \bar{a}_1^*, \bar{a}_2^*\}$ are messy functions of the exogenous parameters of the model—shows $|q_{i,m}^e - q_{i,m}^*| < |q_{i,m}^e - \bar{q}_{i,m}^*|$ for $\delta > 0$ and vice versa for $\delta < 0$ when $s_{i,1} = s_{i,2} = s$.

The example suggests that it might be more essential for agents to display their joint preferences when goods are complementary. It underlines how important it is to take the nature of goods into account when designing markets.

The second example allows for different realizations of the type $s_{i,1} \neq s_{i,2}$. The comparison is less straightforward, but a similar reasoning applies. To achieve tractability, I impose $\mu_s = 0$.

Example 2: Let $\rho^* \in (-1, 1)$, $\mu_{s_1} = \mu_{s_2} = (1/n) \sum_i s_{i,1} = (1/n) \sum_i s_{i,2} \equiv 0$.

An agent $\{s_{i,1}, s_{i,2}\}$ is better off in the disconnected market if and only if

$$TS_i^* - \overline{TS}_i^* > 0 \Leftrightarrow \delta \times \left\{ A(n, \lambda, \delta, \rho^*) s_{i,1} s_{i,2} - B(n, \lambda, \delta, \rho^*) (s_{i,1}^2 + s_{i,2}^2) \right\} > 0,$$

where $A(n, \lambda, \delta, \rho^*) = 2\lambda^3 n^2 + 6\delta\lambda^2 n^2 \rho^* + \delta^2 \lambda (2(n-2)(3n-2) + (16n-8)\rho^{*2}) + \delta^3 ((2n^2-8)\rho^* + 8\rho^{*3})$, $B(n, \lambda, \delta, \rho^*) = \lambda^3 n^2 \rho^* + \delta^3 ((n-2)^2 + 4(n-1)\rho^{*2}) + \delta\lambda^2 (n(3n-4) + 4n\rho^{*2}) + \delta^2 \lambda \times ((3n^2-4)\rho^* + 4\rho^{*3})$.

The agent may obtain a higher total surplus under either the connected or disconnected market structure. This depends on the relative sizes of the interdependence of the goods (δ relative to λ), the correlation of prices (ρ^*), the number of traders in the market (n), and the agents' types $\{s_{i,1}, s_{i,2}\}$.

As in Example 1, the agent is better off in the market structure in which he achieves an amount that is close to what he would achieve if all agents were price-takers. The formulas for $q_{i,m}^*$, $\bar{q}_{i,m}^*$, and $q_{i,m}^e$ are unchanged. With different realizations of types $s_{i,1} \neq s_{i,2}$, however, there are two opposing effects that determine whether $|q_{i,m}^e - q_{i,m}^*|$ is less than or greater than $|q_{i,m}^e - \bar{q}_{i,m}^*|$. The first effect works through the weight that is put on the type for the other good $-m$, i.e., via a_2^* and \bar{a}_2^* . It always works in favor of the connected market. Loosely speaking, it demonstrates that agents can more freely display their joint preferences when the market is connected. The second effect works through how strongly the type for good m determines how much the agent wins of this good, i.e., via a_1^* and \bar{a}_1^* . It could go in favor of either the disconnected or connected market.

When the correlation of clearing prices is sufficiently extreme, causing a_1^* to be sufficiently larger than \bar{a}_1^* , the second effect goes in favor of the disconnected market and can dominate the first effect, which is always in favor of the connected market. Intuitively, traders with favorable realization of types may manage to manipulate the market outcome to their advantage when they are able to make sufficiently precise predictions about how much they will win and pay in the other market, thanks to strong correlations of the clearing prices.

Welfare.—At first sight, one would expect connected markets, in which agents can freely maximize their gains from trade, to achieve a higher welfare than disconnected markets. This intuition is correct when agents are price-takers. In principle, it might fail when they are strategic. Now there might be a trade-off coming from the second effect that is illustrated in Example 2: giving agents more choice of how they display their joint preferences by connecting markets might offer them better ways to manipulate the market's outcome to their personal advantage. This distortive, strategic effect could work in favor of disconnected markets. However, the following stylized example suggests that this is not so common.

Example 3: Let $\rho^* \in (-1, 1)$, $\mu_{s_1} = \mu_{s_2} = (1/n) \sum_i s_{i,1} = (1/n) \sum_i s_{i,2} \equiv 0$. The connected market Pareto dominates the disconnected market, in that $\overline{TS}_i^* \geq TS_i^*$ for all i , if market-clearing prices are sufficiently mildly correlated:

(9) ρ^* such that $\rho^- < \rho^* < \rho^+$ for $\delta > 0$ and $\rho^+ < \rho^* < \rho^-$ for $\delta < 0$, with

$$\rho^- \equiv \frac{-(\delta^2 + \lambda^2)n - \sqrt{\delta^4 n^2 + \lambda^4 n^2 + 2\delta^2 \lambda^2 (8 + (n - 8)n)}}{4\delta\lambda} \quad \text{and}$$

$$\rho^+ \equiv \frac{-(\delta^2 + \lambda^2)n + \sqrt{\delta^4 n^2 + \lambda^4 n^2 + 2\delta^2 \lambda^2 (8 + (n - 8)n)}}{4\delta\lambda}.$$

When correlation is sufficiently mild, no trader strictly prefers the disconnected market. All strategic agents obtain a weakly higher total surplus in the connected market, in which they can express their joint demands. The connected market Pareto dominates the disconnected market. Intuitively, agents no longer predict with sufficient precision how much they will trade in the disconnected market in order to sufficiently manipulate the clearing prices to their favor.

In expectation, the connected market is always more efficient than the disconnected market, as established in the following proposition. It compares how much welfare we expect the two market structures to generate from an ex ante perspective, which is similar to the common practice of comparing expected—rather than realized—revenues of single-sided auctions. The result should be interpreted with some caution, as it might not generalize to settings in which agents have common values for the assets or play nonlinear strategies.

PROPOSITION 5: *Consider markets without noise traders ($\mu_{Q_1} = \mu_{Q_2} = \sigma_Q$). The connected market is expected to achieve higher welfare than the disconnected market:*

$$E[\overline{\mathbf{TS}}^*] - E[\mathbf{TS}^*] = f(n, \lambda, \delta, \rho^*) \times \sigma_s^2 > 0, \quad \text{with}$$

$$f(n, \lambda, \delta, \rho^*) = \frac{4\delta^2(n-2)n(\lambda + \delta\rho^*)(1 - \rho^{*2})}{(\lambda^2 - \delta^2)(n-1)(\lambda^2 n^2 + 2\delta\lambda n^2 \rho^* + \delta^2((n-2)^2 + 4(n-1)\rho^{*2}))}.$$

Before drawing types, all agents are identical in my setting. None of them expects to be able to manipulate the outcome to his own advantage. While some might gain and some might lose ex post, any individual benefit is expected to wash out in the aggregate.

This no longer holds when there are noise traders. From Corollary 5, we know that strategic agents realize a higher total surplus when competing for substitutes in perfectly positively correlated markets. By continuity, this carries over to markets with close to perfect correlation. The next proposition shows that strategic traders may also prefer the market to be disconnected from the ex ante perspective.

PROPOSITION 6: *Strategic agents expect a higher total surplus in the disconnected market than the connected market, i.e., $E[\mathbf{TS}^*] > E[\overline{\mathbf{TS}}^*]$, if*

$$\delta \left[\frac{\rho^*(\mu_{Q_1}^2 + \mu_{Q_2}^2) - 2\mu_{Q_1}\mu_{Q_2}}{n(n-2)} + \frac{2(n-1)(\rho^* - \rho_Q)}{n(n-2)} \sigma_Q^2 + g(n, \lambda, \delta, \rho_s, \rho^*) \sigma_s^2 \right] > 0$$

$$\text{with } g(n, \lambda, \delta, \rho_s, \rho^*) = 4n((n-2)/(n-1)) \left((\lambda + \delta\rho^*) / (\lambda^2 - \delta^2) \right) \\ \times \left[\lambda^3 n^2 (-\rho^* + \rho_s) + \delta^2 \lambda (\rho^* (4 - 3n^2 - 4\rho^{*2}) + (4 - 4\rho^{*2} + n(-8 + 3n + 8\rho^{*2}))) \rho_s \right) + \delta \lambda^2 n (4 - 4\rho^{*2} + 3n(-1 + \rho^* \rho_s)) + \delta^3 (-4n(-1 + \rho^{*2}) + n^2(-1 + \rho^* \rho_s) + 4(-1 + \rho^{*2})(1 + \rho^* \rho_s)) \right] / \left[(\lambda n - \delta(n - 2(1 + \rho^*)))^2 \right. \\ \left. \times (\lambda n + \delta(n - 2(1 - \rho^*)))^2 \right].$$

The proposition provides a sufficient condition for when strategic traders expect a higher total surplus in the disconnected market than the connected market. The expression is equivalent to $E[\mathbf{TS}^*] - E[\overline{\mathbf{TS}}^*]$. This difference decomposes into three parts. The first depends on the means of the total supply, $\{\mu_{Q_1}, \mu_{Q_2}\}$, the second on the variance, σ_Q . The last is driven by the variance of the types, σ_s . It is multiplied by $g(n, \lambda, \delta, \rho_s, \rho^*)$, which depends nontrivially on $\{n, \lambda, \delta, \rho_s, \rho^*\}$. This term makes it difficult to derive additional formal statements, but I conjecture that the condition holds for goods that are sufficiently positively correlated substitutes, i.e., when δ and ρ^* are sufficiently positive.

Overall, the proposition highlights that prices in the disconnected market might allow strategic traders to extract higher rents from nonstrategic traders in expectation.

C. Large Markets

When the market grows large, agents become truthful (Corollary 4). Being truthful, agents no longer need to collect information about the goods in order to purchase or sell the correct amounts. Any informational advantage that might be present in connected markets disappears. As a result, all allocated units go to those agents who truly value them most independent of the market's structure. One achieves the fully efficient allocation, i.e., the allocation of quantities $\{q_{i,1}, q_{i,2}\}_{i=1}^n$ that maximizes the aggregate utility of all agents $\sum_i U(q_{i,1}, q_{i,2}, \bar{s}_i)$ and clears both markets $\sum_i q_{i,m} = Q_m$.

COROLLARY 7: *Either market structure approaches the fully efficient allocation as $n \rightarrow \infty$.*

This efficiency result is not surprising for connected markets. Here we know from previous literature that a uniform-price auction for a single good generates the efficient allocation when no agent has a price impact. This result carries over to the multigood setting (also shown by Malamud and Rostek 2017). It is less trivial for disconnected markets. This is because each market operates under an “auction format [that] is [generally] not rich enough to allow players to express their true [combinatorial] valuations” (Feldman et al. 2016, 8).

While this efficiency result is insightful, it should not be taken for granted in market environments, in which (informed) price-taking agents face additional exogenous noise—for instance, coming from noise traders. More specifically, I conjecture that the efficiency breaks down in markets with infinitely many strategic agents and infinitely many noise traders.²¹

²¹ Similarly, Feldman et al. (2016) warns that the efficiency result is sensitive to the type of noise in the auctions. When total supply is random, it argues, by means of a counterexample, that players might not be able to decide which items to target when they do not know which good will have higher supply. Such “search frictions” may not vanish in the limit.

V. Ideas for Future Research

I conclude by discussing some simplifying assumptions of the model that are relatively strong. Relaxing them is a particularly promising avenue for future research on disconnected markets.

Independent Private Types.—There are two ways to relax this assumption: types could be correlated or interdependent across agents. Prior research provides equilibria for the connected market. Equilibria for disconnected markets are still missing. A starting point on how to derive them could be the proof that derives the equilibrium in my setting (see online Appendix A2). To illustrate how to modify my proof, I derive an equilibrium for a setting with correlated types in online Appendix E. A similar approach could be taken to allow for interdependent values of a simple (linear) form: $\alpha s_{m,i} + (1 - \alpha) \sum_{j \neq i} s_{m,i}$, where $0 \leq \alpha \leq 1$. The equilibrium bidding functions are extremely messy and therefore difficult to work with. To achieve tractability, one could simplify some elements of my framework, depending on the research question.

Ex Ante Symmetric Market Environments.—My analysis focuses on ex ante symmetric market environments. Here I give only one example. The variance of types and total supply is identical across goods. My results therefore do not speak to markets that offer substitutable or complementary securities under highly asymmetric volatility.

With asymmetric volatility of types and total supply across goods, the linear BNE of disconnected markets, which is symmetric across agents, might be asymmetric across goods. More specifically, agents would submit different bidding functions for goods when the variance of how much they win at market clearing $\sigma_{q_m}^2$ differs across markets. An asymmetric equilibrium in the disconnected market then stands in contrast to the unique linear equilibrium of the connected market, which we know to be symmetric across goods. How much agents trade and pay no longer coincides across market structures. Instead, connected and disconnected markets will always generate different sets of allocations.

Whether market designers should be concerned about market connectivity therefore depends not only on the endogenous correlation but also on the endogenous variance of market-clearing prices. While substitutable securities are typically traded in markets with similar volatilities, markets of complementary assets might display highly asymmetric degrees of volatility. From here, one might (once more) conclude that we should be more concerned about connecting markets that offer complementary securities than those that offer substitutable securities.

VI. Conclusion

I investigate why most financial markets are disconnected because traders cannot place orders that are contingent on the prices of bundles of securities, and I analyze the welfare effects of connecting these markets by allowing for such orders. A series of theorems identifies three factors that are crucial when deciding whether to

connect disconnected markets: the correlation of prices, the size of the market, and whether traders have price impact or are price takers. In particular, I show that both market structures achieve the same quantity allocation, and with it the same degree of efficiency, if and only if prices are perfectly correlated or the market is large so that all are price-takers. When noise traders are active in the market, the associated market-clearing prices may differ in a way that allows strategic traders to extract higher rents from them when the market is disconnected. This result generalizes to markets whose prices exhibit imperfect correlations, but only in expectation.

Further, I demonstrate that some strategic agents obtain a higher surplus in the connected market because they can better manipulate the market-clearing price to their advantage when the bidding language is more flexible. This welfare effect might work in favor of disconnected markets. The connected market (ex post) Pareto dominates the disconnected market if prices are sufficiently weakly correlated. In addition, it generates higher welfare in expectation.

One promising avenue for future research would be to deepen the equilibrium analysis for disconnected markets. Previous research provides little guidance on trading dynamics that extend across markets, especially when agents decide *how much* they would like to trade. To simplify the analysis, it is often assumed that agents trade a single unit of an asset. The single-unit demand assumption, however, as been shown to be restrictive in the literature on multiunit auctions. My equilibrium analysis and its extensions and proofs might inspire future theoretic work in this direction.

A different path for future research is to analyze aspects of disconnected and connected markets empirically, using data on market transactions or by means of an experiment. If data on individual behavior, such as bidding data, are available, the necessary condition I derive can be used to back out the true valuations of strategic agents and perform counterfactual analyses. If only market-level data are available, my results could guide an empirical analysis to assess whether and to what extent the correlation of market prices affects measures of market performance—for instance, trade volume.

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