American Economic Journal: Microeconomics 2022, 14(1): 164–185
https://doi.org/10.1257/mic.20180386

Deduction Dilemmas: The Taiwan Assignment Mechanism†

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This paper analyzes the Taiwan mechanism used nationwide for high school assignment starting in 2014. In the Taiwan mechanism, points are deducted from an applicant's score, with larger penalties for lower-ranked choices. Deduction makes the mechanism a hybrid of the Boston and deferred acceptance mechanisms. Our analysis sheds light on why Taiwan's new mechanism has led to massive nationwide demonstrations and why it nonetheless remains in use. (*JEL* D47, I21, I28)

In June 2014, more than five hundred parents marched in Taipei, protesting Taiwan's new mechanism for high school placement. Protestors held placards stating "fill out the preference form for us" and decried admissions as "gambling" (I-chia 2014). Due to this pressure and calls for his resignation, Education Minister Chiang Wei-Ling subsequently issued a formal public apology for the new high school assignment system (CNA 2014b).

What were the protestors complaining about, and why is there so much turmoil associated with Taiwan's new system? To provide insights in response to these questions, this paper analyzes properties of Taiwan's assignment mechanism, a new (to our knowledge) assignment mechanism, which represents a hybrid between the widely studied deferred acceptance and Boston mechanisms.

Taiwan, like many other countries and regions, has recently launched a series of reforms to standardize and centralize its secondary school system. At the turn of the century, rising Taiwanese high schoolers took an admission exam consisting of five subjects. Students submit rankings over schools, and those with a higher score

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†Go to https://doi.org/10.1257/mic.20180386 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

Table 1—Deduction Rules and Number of Applicants across Large Taiwanese Districts in 2014

District	Applicants (1)	Deduction Rule (2)	Total score (3)	Number of options (4)	Max. number of rankings (5)	School × major (6)
Gaoxiong	25,752	0,2,4,6,8,10,12	100	52	7	N
Hualian	3,466	0,4,8,12,16	100	45	5	Y
Jiayi	7,484	0,0,1,1,2,2,3,3,4,4,5,5,5,5,5	87	86	15	Y
Jibei	62,694	0,1,2,3,4,5,6,7,8,9,10,10,10,10,10,10,10,10,10,10,12, 12,12,12,12,12,12,12,12,12	90	139	30	N
Jinmen	652	No deductions	46	12	12	Y
Penghu	869	0,1,2,3,4,5,5,5,5,5,5,5,5,5	78	14	14	Y
Pingdong	7,238	0,2,4,5,6,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7	79	75	30	Y
Taidong	2,270	0,3,6,9,12,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15, 15,15,15,15,15,15,15,15,15,15	100	38	30	Y
Tainan	17,383	0,3,4,5,6	100	48	5	N
Taolian	24,001	0,0,3,3,6,6,9,9,12,12,15,15,15,15,15,15,15,15,15,15,15, 15,15,15,15,15,15,15,15,15	100	34	30	N
Yilan	4,631	No deductions	48	40	N/A	Y
Yunlin	5,829	0,0,0,0,2,2,2,2,4,4,4,4,6,6,6,6,9,9,9,9	90	68	20	Y
Zhanghua	13,465	0,0,2,2,4,4,6,6,8,8,10,10,10,…,10	90	101	N/A	Y
Zhongtou	36,333	0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20, 21,22,23,24,25,26,27,28,29,30,30,30,30,30,30,30,30, 30,30,30,30,30,30,30,30,30,30,30,30	100	63	50	N
Zhumiao	15,561	0,0,0,3,3,3,6,6,6,9,9,9,10,10,10	100	39	15	N

Notes: Compiled by the authors from online sources. Deduction points are listed up to the number of choices an applicant can rank. An ellipsis indicates that all remaining choices have the same deduction. The number of options counts public and vocational schools or school-majors in the choice set. Options that are school-major combinations are indicated in column 6.

chose first. Beginning in the next decade, an essay component was added to the admission exam. Local districts were free to use performance measures aside from exams (such as in Chinese, English, music, or sports) and to choose how to convert these measures to a total score.

In 2014, Taiwan passed the Senior High School Education Act, which established the Comprehensive Assessment Program for Junior High School Students. This act changed how each of the five subjects on the admission exam were scored, placing them into discrete categories: excellent, basic (pass), and needs more work (not pass). To separate high-performing students, excellent scores were further split into A, A+, and A++. During 2014, more than 200,000 pupils took the Comprehensive Assessment Exam and applied to schools in their district of residence.

Besides changing the admission criteria, Taiwanese authorities also changed the assignment mechanism, introducing a deduction system. Loosely speaking, in Taiwan's deduction system, a student's admission score is reduced when a school is ranked lower. Table 1 lists the deduction rule used in several large Taiwanese districts in its first year. In Jibei, where roughly 60,000 students applied in 2014, the system reduces a student's score at her second-choice school by 1, at her third choice by 2, and so on. In Yunlin, the system deducts no points for the first four choices, and the system deducts two points from choices five through eight.[1] This deduction system represents a new class of matching mechanisms, which we term Taiwan mechanisms.

There are many signs that the deduction system is one of the major reasons for nationwide protests. A *China Post* editorial (Wei 2014) states:

> It is outrageous that the students have to have points deducted from their scores because they fill out the wrong slots; it is because of this that many students with A+ in all subjects eventually have to go to the same school with those who have achieved lower scores.

Despite calls to adopt a system where "students can choose the school they want according to their results" (Wei 2014), senior Taiwanese leadership has kept the deduction system for at least six years with only slight modifications, shown in Table 2.

In this paper, we first compare the mechanisms in the Taiwanese class based on vulnerability to manipulation. In this comparison, we use the criteria developed by Pathak and Sönmez (2013). We show that a mechanism becomes more manipulable as we increase deduction points. Then, we analyze the equilibrium properties of the mechanisms in the Taiwanese class. We allow some students to be sincere and some students to be sophisticated (see Pathak and Sönmez 2008). We first show that for any problem, each Taiwan mechanism induces a unique Nash equilibrium outcome that is Pareto efficient. We prove that each student benefits from becoming sophisticated in equilibrium. Finally, we report on some simulations to analyze the winners and losers of a possible shift from the Taiwan mechanism to a nonmanipulable mechanism.

As far as we know, the only other paper to study Taiwan's new system is Hsu (2014), who studies the new admission test. Our work is most closely related to Ergin and Sönmez (2006) and Pathak and Sönmez (2008). Both papers consider the equilibrium of the preference revelation game induced by the Boston mechanism under different assumptions about player sophistication. The results are related because the Taiwan mechanism generalizes aspects of the Boston mechanism. Chen and Kesten's (2017) study of Chinese college admissions is also related, since the permanency-execution period in China is related to deduction. Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003) initiated the formal research on the mechanism-design approach to student assignment. Pathak (2016) surveys this literature.

In the next section, after introducing the model and formal definitions, we examine the incentive properties of Taiwan mechanisms, showing that they can be compared in terms of manipulation based on a natural ordering of their deduction points. In Section II, we analyze the equilibrium of the preference revelation game induced by the Taiwan mechanism and compare it with the deferred acceptance algorithm and provide the results of the simulation analysis. The last section concludes.

[1] In some accounts, deduction involves adding points to higher-ranked choices. This is identical to deducting points from lower-ranked choices.

Table 2—Changes to Deduction Rules

District	Year of change	From	To
Gaoxiong	2015	0,2,4,6,8,10,12	0,0,0,2,2,2,4,4,4,4,4,4,4,4,4,4,4,4,4,4
	2016	0,0,0,2,2,2,4,4,4,4,4,4,4,4,4,4,4,4,4,4	0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2
Hualian	2015	0,4,8,12,16	0,0,0,2,2,2,4,4,4,6,6,6,8,8,8,10,10,10,12,12,12,14,14,14,16,16,16,18,18,18,20,20,20,20,20,20,20,20,20,20
Jiayi	2015	0,0,1,1,2,2,3,3,4,4,5,5,5,5,5	0,0,0,2,2,2,4,4,4,6,6,6,8,8,8,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10
	2016	0,0,0,2,2,2,4,4,4,6,6,6,8,8,8,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10	0,0,0,0,0,0,1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,3,3,4,4,4,4,4,4
Jibei	2015	0,1,2,3,4,5,6,7,8,9,10,10,10,10,10,10,10,10,10,10,12,12,12,12,12,12,12,12,12,12	0,0,0,0,0,1,1,1,1,1,2,2,2,2,2,3,3,3,3,3,4,4,4,4,4,4,4,4,4,4
Jinmen	2015	No deductions	0,2,4,5,6,6,6,6,7,7,7,7
	2016	0,2,4,5,6,6,6,6,7,7,7,7	0,1,2,3,4,4,4,4,5,5,5,5
Pingdong	2015	0,2,4,5,6,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7	0,0,0,2,2,2,4,4,4,5,5,5,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6
Taidong	2015	0,3,6,9,12,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15	0,3,6,9,12,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18,18
Tainan	2015	0,3,4,5,6	0,2,3,4,5,10, . . . ,10
	2016	0,2,3,4,5,10, . . . ,10	0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5, . . . ,5
Taolian	2017	0,0,0,3,3,3,6,6,6,9,9,9,12,12,12,15,15,15,15,15,15,15,15,15,15,15,15,15,15,15	0,0,0,3,3,3,6,6,6,9,9,9,12,12,12,14,14,14,14,14,14,14,14,14,14,14,14,14,14,14
Yilan	2018	No deductions	0,0,0,0,2,2,2,2,4,4,4,4,6,6,6,6,9,9,9,9
Yunlin	2016	0,0,0,0,0,0,1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,3,3,4,4,4,4,4,4	0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2,2,2,3,3,3,3,3,3,3,3,3,3,4,4,4,4,4,4,4,4,4,4
Zhanghua	2015	0,0,2,2,4,4,6,6,8,8,10,10,10, . . . ,10	0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5,6,6,6,7,7,7,8,8,8,9,9,9,10,10,10,11,11,11,12,12,12,13,13,13,14,14,14,15,15,15
	2016	0,0,0,1,1,1,2,2,2,3,3,3,4,4,4,5,5,5,6,6,6,7,7,7,8,8,8,9,9,9,10,10,10,11,11,11,12,12,12,13,13,13,14,14,14,15,15,15	0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
Zhongtou	2015	0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,30,30,30,30,30,30,30,30,30,30,30,30,30,30,30,30,30,30,30	0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2
Zhumiao	2015	0,0,0,3,3,3,6,6,6,9,9,9,10,10,10	0,0,0,3,3,3,5,5,5,7,7,7,9,9,9

Notes: Compiled by the authors from online sources. Deduction points are listed up to the number of choices an applicant can rank. An ellipsis indicates that all remaining choices have the same deduction.

I. Model

A. *Primitives*

In a school choice problem, there is a finite set of students and schools, each with maximum capacity. Each student has a preference order over all schools and remaining unassigned, and a priority score at each school.

Formally, a school choice problem includes

(i) students $I = \{i_1, \ldots, i_n\}$,

(ii) schools $S = \{s_1, \ldots, s_m\}$,

(iii) capacity vector $q = (q_{s_1}, \ldots, q_{s_m})$,

(iv) a list of strict student preferences $P = (P_{i_1}, \ldots, P_{i_n})$, and

(v) a list of strict school priority score profiles $\pi = (\pi_{s_1}, \ldots, \pi_{s_m})$.

For any student i, P_i is a strict preference relation over $S \cup \{\varnothing\}$, where $\varnothing$ denotes being unassigned and $s\ P_i\ \varnothing$ means student i considers school s acceptable.[2] For any student i, let R_i denote the "at least as good as" relation induced by P_i. We denote the rank of school s under P_i with $r_s(P_i)$, i.e., $r_s(P_i) = |\{s' \in S \cup \{\varnothing\} : s'\ P_i\ s\}| + 1$. For any school s, the function $\pi_s : I \rightarrow \mathbb{R}_+$ is school s's priority score profile such that $\pi_s(i) = \pi_s(j)$ if and only if $i = j$. Here, $\pi_s(i) > \pi_s(j)$ means that student i has higher priority than student j at school s. The priority order of school s over students implied by π_s is $\succ_s^\pi$, i.e.,

$$i \succ_s^\pi j \quad \Leftrightarrow \quad \pi_s(i) > \pi_s(j).$$

Let $\pi_{\max}$ be the maximum possible score.

We fix the set of students and schools and capacity throughout the paper, so (P, π) denotes a school choice problem. The outcome of a school choice problem is a **matching** $\mu : I \rightarrow S \cup \{\varnothing\}$, or a function such that $|\mu^{-1}(s)| \leq q_s$ for any school $s \in S$. The assignment of student i under matching μ is denoted with $\mu(i)$. With slight abuse of notation, we use $\mu(s)$ instead of $\mu^{-1}(s)$ in the rest of the paper.

A matching μ is **Pareto efficient** if there is no other matching ν such that $\nu(i)\ R_i\ \mu(i)$ for all $i \in I$ and $\nu(j)\ P_j\ \mu(j)$ for some $j \in I$. A matching μ is **individually rational** if there is no student i such that $\varnothing\ P_i\ \mu(i)$. A matching μ is **nonwasteful** if there is no student-school pair (i, s) such that $s\ P_i\ \mu(i)$ and $|\mu(s)| < q_s$. A matching μ is **fair** if there is no student-school pair (i, s) such that $s\ P_i\ \mu(i)$ and $\pi_s(i) > \pi_s(j)$ for some $j \in \mu(s)$.

A matching μ is **stable** if it is individually rational, nonwasteful, and fair. It is well known that there exists a stable matching that is weakly preferred to any stable matching by each student (Gale and Shapley 1962). We refer to this matching as the **student-optimal stable matching**.

A **mechanism**, denoted by φ, is a systematic procedure that selects a matching for each problem. Let $\varphi(P, \pi)$ denote the matching selected by φ in problem (P, π), $\varphi(P, \pi)(i)$ denote the assignment of student i, and $\varphi(P, \pi)(s)$ denote the set of students assigned to school s. We say a mechanism φ is Pareto efficient (nonwasteful) [stable] ⟨individually rational⟩ if $\varphi(P, \pi)$ is Pareto efficient (nonwasteful) [stable] ⟨individually rational⟩ for any (P, π).

A mechanism φ is **vulnerable to manipulation** in (P, π) if there exists i and P_i' such that

$$\varphi(P_i', P_{-i}, \pi)(i)\ P_i\ \varphi(P, \pi)(i),$$

where $P_{-i} = (P_j)_{j \neq i}$. A mechanism φ is **strategy-proof** if there is no problem for which it is vulnerable to manipulation.

[2] Hereafter, we consider $\varnothing$ as a "null" school with $q_\varnothing = |I|$.

B. *Taiwan Mechanisms*

Taiwan mechanisms are a hybrid between student-proposing deferred acceptance and Boston mechanisms. To define a Taiwan mechanism, we first describe these two mechanisms.

Given (P, π), the student-proposing **deferred acceptance (DA)** mechanism computes its outcome as follows:

Step 1: Each student i applies to her best choice, possibly $\varnothing$, according to P_i. Each school s tentatively accepts the best q_s students among all applicants according to π_s and rejects the rest.

⋮

Step $k > 1$: Each student i applies to her best choice which has not yet rejected her, possibly $\varnothing$, according to P_i. Each school s tentatively accepts the best q_s students among all applicants according to π_s and rejects the rest.

The algorithm terminates when there are no more rejections. Students are assigned to the choices they have applied to in the last step.

Given (P, π), the **Boston mechanism (BM)** computes its outcome as follows:

Step 1: Each student i applies to her best choice, possibly $\varnothing$, according to P_i. Each school s permanently accepts the best q_s students among all applicants according to π_s and rejects the rest. Each accepted student and her assigned seat are removed.

⋮

Step $k > 1$: Each remaining student i applies to her kth choice, possibly $\varnothing$, according to P_i. Each school s permanently accepts the best students among all applicants according to π_s up to the number of its remaining seats and rejects the rest. Each accepted student and her assigned seat are removed.

The algorithm terminates when there are no more rejections.

A Taiwan mechanism can be implemented by deducting points from student priority scores and then applying the DA to the resulting problem. We define a **deduction rule** as

$$\lambda = \left(\lambda_1, \lambda_2, \ldots, \lambda_{|S|+1}\right) \in \mathbb{R}_+^{|S|+1}$$

such that $\lambda_1 = 0$ and $\lambda_k \leq \lambda_{k+1}$ for any $k \in \{1, 2, \ldots, |S|\}$. For any two deduction rules λ and λ', if $\lambda_k \geq \lambda'_k$ for all $k \in \{1, 2, \ldots, |S| + 1\}$ and $\lambda_{k'} > \lambda'_{k'}$ for some $k' \in \{1, 2, \ldots, |S|\}$, then $\lambda > \lambda'$.

The **Taiwan mechanism** associated with deduction rule λ is denoted with TM^{λ}. For problem (P, π), the outcome of TM^{λ} is simply $DA(P, \hat{\pi}^{\lambda})$, where for each student-school pair (i, s),

$$\hat{\pi}_s^{\lambda}(i) = \pi_s(i) - \lambda_{r_s(P_i)}.$$

When deduction points are zero, the associated Taiwan mechanism produces the same outcome as the DA. That is, if $\lambda^1 = (0, 0, \ldots, 0)$, then

$$TM^{\lambda^1}(P, \pi) = DA(P, \pi).$$

When deduction points are very large, the Taiwan mechanism produces the same outcome as the BM. That is, if $\lambda^2 = (0, \bar{\pi}, 2\bar{\pi}, \ldots, |S|\bar{\pi})$ where $\bar{\pi} > \pi_{\max}$, then

$$TM^{\lambda^2}(P, \pi) = BM(P, \pi).$$

Since the Taiwan mechanism can be implemented as the DA with different inputs, it inherits some properties of the DA. Since the DA is nonwasteful and individually rational, $TM^{\lambda}(P, \pi)$ is nonwasteful and individually rational for any λ and (P, π).

Through the deduction rule, Taiwan mechanisms can produce the same outcomes as a large number of other mechanisms. Aside from DA and BM, the First Preference First mechanisms outlawed in England described by Pathak and Sönmez (2013) are in the class of Taiwan mechanisms if the deduction rule can depend on the school. Since the Chinese Parallel mechanisms described by Chen and Kesten (2017) span the DA and BM extremes, they can also be represented as Taiwan mechanisms.[3,4]

We make two assumptions throughout the rest of the analysis. Since we are motivated by Taiwanese policy developments, we assume that all schools share the same strict priority score profiles.

ASSUMPTION 1: *For all* $s, s' \in S$, $\pi_s = \pi_{s'}$.

[3] In particular, a deduction rule would be related to the permanency-execution period in the Chinese Parallel mechanism. If the choices within an execution period all have the same deduction points, and the deduction points in an earlier block are all sufficiently larger than those in a later block, then such a deduction rule produces the same outcome as the Chinese Parallel mechanism.

[4] Another example of an assignment system in which students' submitted preference lists affect the way colleges rank them is the Joint University Programmes Admissions System (JUPAS) in Hong Kong (Liu and Chiu 2011). In JUPAS, colleges first actively rank students. Colleges might have different base rankings of students. After students submit their preference lists, colleges are informed how the students have ranked them. Given this information, colleges may update their base rankings by favoring the students ranking them higher.

In Taiwan, a student priority score is determined by a combination of measures including test scores. It is the same at all schools in a district.[5] We also assume that there are no ties in the deducted priority scores.

ASSUMPTION 2: *For any preference profile P, there are no ties in the deducted priority after applying the deduction rule λ to the problem (P,π). That is, for all P and $s \in S$,*

$$\hat{\pi}_s^{\lambda}(i) = \hat{\pi}_s^{\lambda}(j) \quad \Rightarrow \quad i = j.$$

To understand the properties of Taiwan mechanisms, consider the following example.

Example 1: There are four schools, $S = \{a,b,c,d\}$, each with one seat, and four students, $I = \{i_1,i_2,i_3,i_4\}$. The student priority scores for each school $s \in S$ are: $\pi_s(i_1) = 100$, $\pi_s(i_2) = 50$, $\pi_s(i_3) = 11$, and $\pi_s(i_4) = 0$. Let $\pi_{\max} = 100$. The preferences of the students are as follows:

P_{i_1}:	a	b	c	d	$\varnothing$
P_{i_2}:	a	b	c	d	$\varnothing$
P_{i_3}:	b	c	d	a	$\varnothing$
P_{i_4}:	c	a	d	b	$\varnothing$

Consider two different deduction rules: $\lambda^1 = (0,41,45,51,51)$ and $\lambda^2 = (0,110,220,330,440)$. The corresponding priority orders for π, $\hat{\pi}^{\lambda^1}$, and $\hat{\pi}^{\lambda^2}$ are as follows:

	π				$\hat{\pi}^{\lambda^1}$				$\hat{\pi}^{\lambda^2}$			
a:	i_1	i_2	i_3	i_4	i_1	i_2	i_3	i_4	i_1	i_2	i_4	i_3
b:	i_1	i_2	i_3	i_4	i_1	i_3	i_2	i_4	i_3	i_1	i_2	i_4
c:	i_1	i_2	i_3	i_4	i_1	i_2	i_4	i_3	i_4	i_3	i_1	i_2
d:	i_1	i_2	i_3	i_4	i_1	i_2	i_3	i_4	i_3	i_4	i_1	i_2

The table orders applicants at schools from left to right, so that, e.g., $\pi_a(i_1) > \pi_a(i_2) > \pi_a(i_3) > \pi_a(i_4)$.

Under problem (P,π), the matching produced by the DA is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

[5] This assumption is consistent with the tie breaker in use. Online Appendix A presents two examples showing that our main results do not carry over to the environment where priorities may differ across schools.

The matching produced by the Taiwan mechanism with deduction λ^1, i.e., TM^{λ^1}, is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & c & b & d \end{pmatrix}.$$

The matching produced by TM^{λ^2}, which is equivalent to *BM*, is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & d & b & c \end{pmatrix}.$$

C. *Comparing Incentives across Mechanisms*

When deduction points are very large, the Taiwan mechanism is equivalent to the BM, and when deduction points are all zero, it reduces to a serial dictatorship mechanism (which is equivalent to the DA when school priorities are the same across all schools). A serial dictatorship mechanism is a strategy-proof mechanism, while BM is highly manipulable. Does this comparison extend to intermediate values of the deduction rule?

To answer this question, we use the following criteria to compare manipulation possibilities across mechanisms developed by Pathak and Sönmez (2013).

DEFINITION: *Mechanism ψ is more manipulable than φ if*

(i) *in any (P, π) such that φ is vulnerable to manipulation, ψ is also vulnerable to manipulation, and*

(ii) *there exists some (P, π) such that ψ is vulnerable to manipulation, but φ is not.*

If we only know that (i) holds and are not sure whether (ii) holds or not, then we say that ψ is **at least as manipulable as** φ.

Returning to Example 1, consider deduction rule $\lambda^3 = (0, 9, 20, 30, 30)$. Note that $\lambda_k^3 < \lambda_k^1$ for each $k > 1$. The corresponding priority orders for π, $\hat{\pi}^{\lambda^3}$, and $\hat{\pi}^{\lambda^1}$ are as follows:

	π	$\hat{\pi}^{\lambda^3}$	$\hat{\pi}^{\lambda^1}$
a:	i_1 i_2 i_3 i_4	i_1 i_2 i_4 i_3	i_1 i_2 i_3 i_4
b:	i_1 i_2 i_3 i_4	i_1 i_2 i_3 i_4	i_1 i_3 i_2 i_4
c:	i_1 i_2 i_3 i_4	i_1 i_2 i_3 i_4	i_1 i_2 i_4 i_3
d:	i_1 i_2 i_3 i_4	i_1 i_2 i_3 i_4	i_1 i_2 i_3 i_4

The outcome of the Taiwan mechanism with deduction λ^1 is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & c & b & d \end{pmatrix}.$$

If student i_2 instead reports b as her top choice, then she obtains a better outcome than under truth telling. She has a higher score than i_3 and i_4 under $\hat{\pi}_b^{\lambda^1}$ when she ranks b as her top choice. Under TM^{λ^1}, student i_1 never applies to school b. Therefore, at school b, student i_2 is not rejected and is assigned to the more preferred school b when she ranks b as her top choice.

On the other hand, TM^{λ^3} produces

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix},$$

which is the same as DA for (P,π). No student can manipulate TM^{λ^3} in (P,π).

This example illustrates how the potential for manipulation increases as we increase deduction points. Our first proposition shows the pattern holds in general.

PROPOSITION 1: *Under Assumptions 1 and 2, if* $\lambda^1 > \lambda^2$, *then* TM^{λ^1} *is more manipulable than* TM^{λ^2}.

PROOF:

We first show that there exists at least one problem (P,π) such that no student can manipulate TM^{λ^2}, but some students can manipulate TM^{λ^1}. Suppose $\lambda_k^1 = \lambda_k^2$ for all $k < \bar{k}$ and $\lambda_{\bar{k}}^1 > \lambda_{\bar{k}}^2$ for some $\bar{k} \in \{2,\ldots,|S|\}$. Let $S = \{s_1,\ldots,s_{\bar{k}},\ldots\}$, $I = \{i_1,\ldots,i_{\bar{k}+1},\ldots\}$, $|I| \geq \bar{k}+1$, and $q_s = 1$ for all $s \in S$. Student i_k has the kth highest score under π. Student i_k prefers school s_k as the top choice for all $k < \bar{k}$, and student i_k prefers $\varnothing$ as the top choice for all $k > \bar{k}+1$. The preference of $i_{\bar{k}}$ is $s_k \, P_{i_{\bar{k}}} \, s_{k+1} \, P_{i_{\bar{k}}} \, \varnothing$ for all $k \in \{1,\ldots,|S|-1\}$. School $s_{\bar{k}}$ is the only acceptable school for $i_{\bar{k}+1}$. Let $\pi_s(i_{\bar{k}}) - \lambda_{\bar{k}}^1 < \pi_s(i_{\bar{k}+1}) < \pi_s(i_{\bar{k}}) - \lambda_{\bar{k}}^2$. Then, $TM^{\lambda^1}(P,\pi)(i) = TM^{\lambda^2}(P,\pi)(i)$ for all $i \in I \setminus \{i_{\bar{k}}, i_{\bar{k}+1}\}$, $TM^{\lambda^2}(P,\pi)(i_{\bar{k}}) = TM^{\lambda^1}(P,\pi)(i_{\bar{k}+1}) = s_{\bar{k}}$. Then, student $i_{\bar{k}}$ can manipulate TM^{λ^1} by ranking $s_{\bar{k}}$ as their top choice, but no student can manipulate TM^{λ^2}.

Next, we show that TM^{λ^1} is at least as manipulable as TM^{λ^2}. We present two observations and three lemmas that we use in the proof.

OBSERVATION 1: *For any* (P,π), λ, *and* $i \in I$, *if* $s \, P_i \, s'$, *then* $\hat{\pi}_s^{\lambda}(i) \geq \hat{\pi}_{s'}^{\lambda}(i)$.

This follows from the fact that $\lambda_k \leq \lambda_{k-1}$ for any λ.

OBSERVATION 2: *For any* (P,π), *there exists a unique stable matching which is the outcome of the serial dictatorship* (*SD*) *mechanism under* π *and P. Hence*, *the unique stable matching is also Pareto efficient.*

This follows from the fact that $\pi_s = \pi_{s'}$ for any $s, s' \in S$. With slight abuse of notation, we use $\pi(i)$ instead of $\pi_s(i)$ in the rest of the proof.

LEMMA 1: *For an arbitrary* (P,π), *let* μ *be the unique stable matching and* ν *be another matching such that* $\nu \neq \mu$. *Then, there exists a student i such that* $\mu(i) \, P_i \, \nu(i)$ *and* $\mu(j) = \nu(j)$ *for any student j with* $\pi(j) > \pi(i)$.

PROOF:

By Observation 2, μ is Pareto efficient and $\mu = SD(P,\pi)$. Since μ is Pareto efficient, $\nu \neq \mu$ implies that there exists a student i' such that $\mu(i')\ P_{i'}\ \nu(i')$. Without loss of generality, let i be the student with the highest priority score under π who prefers μ to ν. On the contrary, suppose there exists a student j with $\pi(j) > \pi(i)$ and $\mu(j) \neq \nu(j)$. Without loss of generality, let j be such a student with the highest priority score under π. Then, $\nu(j)\ P_j\ \mu(j)$. However, this contradicts μ being the outcome of the SD and i being the highest-scoring student who prefers μ to ν. ∎

We consider the sequential version of the DA defined by McVitie and Wilson (1970), where students apply one at a time according to a predetermined order χ and, in each step, the student who has the highest rank in χ among the ones whose offer has not been tentatively accepted (held) applies.

LEMMA 2: *For arbitrary (P,π), λ, and order χ, consider any step k of the sequential DA under $(P,\hat{\pi}^{\lambda})$ such that there is only one student i who has not been tentatively accepted (held) by some school in $S \cup \{\varnothing\}$. If school s that i applies to in step k tentatively accepts her offer, then i is assigned to s when the sequential DA terminates.*

PROOF:

Let $t_{\bar{k}} = \hat{\pi}^{\lambda}_{\bar{s}}(\bar{i})$ such that student $\bar{i}$ applies in step $\bar{k}$ of the sequential DA to school $\bar{s}$.

In step k of the sequential DA, student i is tentatively accepted by school s if either the number of tentatively accepted students in step $k-1$ by s is less than q_s or there exists a student j who is tentatively accepted (held) in Step $k-1$ by s and $\hat{\pi}^{\lambda}_s(i) > \hat{\pi}^{\lambda}_s(j)$. If the prior case holds, then the mechanism terminates and the desired result follows. If the latter case holds by Observation 1 and the fact that in each future step at most one student is not tentatively accepted (held), then $t_{k'} < \hat{\pi}^{\lambda}_s(i)$ for any $k' > k$. Therefore, i will not be rejected by s. ∎

LEMMA 3: *For arbitrary (P,π) and λ, let $\mu = DA(P,\pi) = SD(P,\pi)$ and $\nu = TM^{\lambda}(P,\pi) = DA(P,\hat{\pi}^{\lambda})$. If $\mu \neq \nu$, then there exists a student who can manipulate TM^{λ} in (P,π).*

PROOF:

First, we define an axiom, known as population monotonicity, that we use throughout the proof.[6] A mechanism φ is population monotonic if for any (I,S,q,P,π) after removal of any student i the assignment of all remaining students are (weakly) improved, i.e., for all $j \in I\setminus\{i\}$,

$$\varphi\big(I\setminus\{i\},S,q,P_{-i},\pi|\big(I\setminus\{i\}\big)\big)(j)\ R_j\ \varphi\big(I,S,q,P,\pi\big)(j),$$

[6] We also use population monotonicity in the proof of Proposition 3.

where $\pi|(I\setminus\{i\})$ is the restriction of π on students in $I\setminus\{i\}$.

By Lemma 1, there exists a student i such that $\mu(i)\ P_i\ \nu(i)$ and $\mu(j) = \nu(j)$ for any student j with $\pi(j) > \pi(i)$. Since TM^{λ} is nonwasteful, there exists a student k such that $\nu(k) = \mu(i)$ and $\pi(i) > \pi(k) \geq \hat{\pi}^{\lambda}_{\mu(i)}(k)$. Under $(P, \hat{\pi}^{\lambda})$, we consider the sequential DA for an order χ such that student i is the last student under χ. First note that, when it is i's turn all seats at $\mu(i)$ are tentatively filled. Otherwise, i would be matched to $\mu(i)$ or better school under ν. By the population monotonicity of the sequential DA, when it is i's turn to apply, there exists at least one student j' who is tentatively accepted by $\mu(i)$ and $\pi(i) > \hat{\pi}^{\lambda}_{\mu(i)}(k) \geq \hat{\pi}^{\lambda}_{\mu(i)}(j')$. This follows from the fact that under the tentative matching attained just before i's turn, student k is assigned to a weakly better school than $\nu(k) = \mu(i)$, and when the sequential DA terminates it selects matching ν. Hence, Lemma 2 implies that student i can get $\mu(i)$ by ranking it as top choice. ∎

To complete the proof of Proposition 1, let μ be the student-optimal stable matching, i.e., $\mu = SD(P,\pi) = DA(P,\pi)$, and ν^1 and ν^2 be the outcomes of TM^{λ^1} and TM^{λ^2}, respectively. By Lemma 3, if $\nu^1 \neq \mu$, then there exists a student j who can manipulate TM^{λ^1}.

We consider two more cases.

Case 1: $\nu^1 = \nu^2 = \mu$. Suppose i is assigned to school s by manipulating TM^{λ^2}. For both $(P, \hat{\pi}^{\lambda^1})$ and $(P, \hat{\pi}^{\lambda^2})$, we consider the sequential DA for an order χ in which i applies last. Let $\bar{\nu}^1$ and $\bar{\nu}^2$ be the tentative allocations obtained just before i's turn for $(P, \hat{\pi}^{\lambda^1})$ and $(P, \hat{\pi}^{\lambda^2})$, respectively. By the fact that $\nu^1 = \nu^2 = \mu$ and Lemma 2 and Observation 1, $\bar{\nu}^1(s') = \bar{\nu}^2(s') = \mu(s')$ and for all $s'\ P_i\ \mu(i)$. Since i can get s by manipulating TM^{λ^2}, there exists a student $\bar{i} \in \bar{\nu}^2(s)$ such that $\hat{\pi}^{\lambda^2}_s(\bar{i}) < \pi(i)$. Then, $\hat{\pi}^{\lambda^2}_s(\bar{i}) < \pi(i)$ and $\lambda^1 > \lambda^2$ imply that $\hat{\pi}^{\lambda^1}_s(\bar{i}) < \pi(i)$. Hence, Lemma 2 implies that i can get s by ranking it as top choice under TM^{λ^1}.

Case 2: $\nu^2 \neq \nu^1 = \mu$. By Proposition 2 (see Section IIA), ν^1 and ν^2 are Pareto efficient under preference profile P. Hence, there exists at least one student k who prefers $\nu^2(k)$ to $\nu^1(k)$. Let $\bar{I} = \{i \in I : \nu^2(i)\ P_i\ \nu^1(i)\}$ and $k \in \bar{I}$ have a higher priority score than all other students in $\bar{I}$, i.e., $\pi(k) > \pi(j)$ for each $j \in \bar{I}\setminus\{k\}$. That is, $\nu^1(i)\ R_i\ \nu^2(i)$ for each $i \in I$ with $\pi(i) > \pi(k)$. Since $\nu^1 = \mu = SD(P,\pi)$, there exists at least one student $\bar{i}$ such that $\pi(\bar{i}) > \pi(k)$, $\nu^2(k) = \nu^1(\bar{i})$, and $\nu^1(\bar{i})\ P_{\bar{i}}\ \nu^2(\bar{i})$. Let $s = \nu^2(k) = \nu^1(\bar{i})$. Then, by the stability of ν^2 under $(P, \hat{\pi}^{\lambda^2})$, we have $\pi(k) \geq \hat{\pi}^{\lambda^2}_s(k) > \hat{\pi}^{\lambda^2}_s(\bar{i})$. By the fact that $\lambda^1 > \lambda^2$, we have $\pi(k) > \hat{\pi}^{\lambda^2}_s(\bar{i}) \geq \hat{\pi}^{\lambda^1}_s(\bar{i})$. Then, for $(P, \hat{\pi}^{\lambda^1})$, we consider the sequential DA mechanism for an order χ in which k applies last. Let $\bar{\nu}^1$ be the tentative allocation obtained just before k's turn for λ^1. By the fact that $\nu^1 = \mu$ and Lemma 2, $\bar{\nu}^1(s') = \mu(s')$ for all $s'\ P_k\ \mu(k)$. Hence, Lemma 2 implies that k can get s by ranking it as their top choice under TM^{λ^1}. ∎

Within the class of Taiwan mechanisms, the BM involves large deduction points.[7] Therefore, Proposition 1 implies the following:

COROLLARY 1: *The BM is more manipulable than any other Taiwan mechanism.*

Proposition 1 relates to a statement of a principal in Taipei who remarked "as long as the deduction system exists, problems cannot be solved" (CNA 2014a). That is, the manipulation possibilities are only eliminated when there is zero deduction.

There have been several changes to deduction rules since the system's first year, in 2014. Table 2 shows that in most cases, districts have relaxed the deduction rules compared to the first year. For instance, in Gaoxiong, each choice has a weakly smaller deduction in 2015 than in 2014. The two largest districts by number of applications, Jibei and Zhongtou, also changed their deduction rules to reduce deduction amounts. Proposition 1 implies that each of these changes have made the mechanism less manipulable. However, not all changes involve moves to less manipulable mechanisms. For instance, in Jinmen, there were no deductions in 2014, while in 2015 there were deductions for non-top choices.

II. Equilibrium Analysis

A. *Characterization*

Next, we analyze the equilibrium properties of Taiwan mechanisms by considering the Nash equilibrium of the simultaneous preference revelation game under complete information induced by a Taiwan mechanism—in this section, the **Taiwan game**.

Following Pathak and Sönmez (2008), we assume there are two types of students. Many families report confusion about the new Taiwanese mechanism, but some have learned about the mechanism's rules. Let $\mathcal{N}$ and $\mathcal{M}$ denote the set of sincere students and sophisticated students, respectively. For each $i \in \mathcal{N}$, the strategy space of student i is $\{P_i\}$, so i can only submit her true preference. This modeling choice captures the fact that some participants may not understand how deductions change their incentives. For each $j \in \mathcal{M}$, student j's strategy space is all strict preferences over schools, including being unassigned.

To understand the properties of equilibrium, we define the augmented priority scores of sincere and sophisticated students as follows:

DEFINITION: *Given a problem* (P, π) *and deduction rule* λ, *construct an* **augmented priority score profile** $\tilde{\pi}$ *as:*

(i) *For each school* s, *adjust the priority score of each sincere student* $i \in \mathcal{N}$ *according to* $\tilde{\pi}_s(i) = \pi_s(i) - \lambda_{r_s(P_i)}$ *(i.e., apply the deduction rule to sincere students for school* s*).*

[7] In particular, if $TM^{\lambda}(P,\pi) \neq BM(P,\pi)$, then there exists $\lambda' > \lambda$ such that $TM^{\lambda'}(P,\pi) = BM(P,\pi)$.

(*ii*) *For each school s, keep the priority score for each sophisticated student* $j \in \mathcal{M}$ *unchanged* $\tilde{\pi}_s(j) = \pi_s(j)$.

In Example 1, the unique Nash equilibrium outcome and the unique stable matching under the augmented priority scores coincide.

Example 1 (continued)**:** Suppose the deduction rule is λ^1, students i_1 and i_3 are sincere, and students i_2 and i_4 are sophisticated. Students i_1 and i_3 will report P_{i_1} and P_{i_3}, respectively. Student i_1 is assigned school a in any equilibrium outcome, independently of the strategies played by the other students. Moreover, student i_2 is assigned school b when she ranks it first, and i_3 is assigned school b when i_2 does not rank b first. Hence, in any equilibrium, i_2 ranks b first and is assigned b. Similarly, when i_2 ranks b first, student i_4 can be assigned school c by only ranking it first.

Hence, there is a unique Nash equilibrium outcome

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & d & c \end{pmatrix}.$$

The augmented priority orders associated with $\tilde{\pi}$ are as follows:

a:	i_1	i_2	i_4	i_3
b:	i_1	i_2	i_3	i_4
c:	i_1	i_2	i_4	i_3
d:	i_2	i_1	i_4	i_3

The unique stable matching under $(P, \tilde{\pi})$ is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & d & c \end{pmatrix},$$

which is the same as the Nash equilibrium outcome.

The observation that the Nash equilibrium outcome is related to the stable matching under the augmented priority score holds generally.

We first show that under problem $(P, \tilde{\pi})$ there exists a unique stable matching.

PROPOSITION 2: *Under Assumptions 1 and 2, for any given* (P, π), λ, $\mathcal{M}$, *and* $\mathcal{N}$, *let* $\tilde{\pi}$ *be the augmented priority score profile. Under* $(P, \tilde{\pi})$ *there exists a unique stable matching, and it is Pareto efficient.*

PROOF:

By using a recursive procedure, under $(P, \tilde{\pi})$, we show that one can always find a student who has the highest priority among the remaining students at her top choice among the remaining schools, and we remove that student and a seat from that school. This student will be matched to her top choice among the remaining schools in any stable matching, and there will not exist a possible welfare improvement trade between the remaining students and this student.

Let i_k be the student who has the highest score in π among the students remaining in Step $k \geq 1$ of this procedure. We start with Step 1, in which all students and all seats are available. By our construction, i_1 has the highest priority at her top choice whether she is sophisticated or sincere. Moreover, she will be assigned to her top choice in any stable matching and, therefore, her welfare cannot be improved by trading with the other students. We remove i_1 and one seat from her top choice and consider the remaining students and schools in Step 2.

Suppose our claim holds for the first $k - 1$ steps of this procedure. Now consider Step k. If i_k is a sophisticated student, then she has the highest priority among the remaining students at all remaining schools. Hence, the claim holds. Suppose i_k is a sincere student. Let s^1 be her top choice among the remaining schools. If the student with the highest priority for s^1 among the remaining students prefers s^1 most among the remaining schools, then we are done. Otherwise, we consider the student $i^1 \neq i_k$ with the highest priority at s^1 among the remaining students and her most preferred school among the remaining students denoted by $s^2 \neq s^1$. Note that i_k cannot have higher priority than i^1 at school s^2. If the student with the highest priority for s^2 among the remaining students prefers s^2 most among the remaining schools, then we are done. Otherwise, we consider the student $i^2 \notin \{i^1, i_k\}$ with the highest priority at s^2 among the remaining students and her most preferred school among the remaining ones denoted by s^3. Note that i_k and i^1 cannot have higher priority than i^2 at school s^3 and $s^3 \notin \{s^1, s^2\}$. By finiteness, we will eventually find a student i and a school s such that i has the highest priority at s among the remaining students and i prefers s most among the remaining schools. Hence, in any stable matching, i is assigned to s and there cannot be a welfare-improving trade involving student i. ∎

Next, we we analyze the equilibrium outcome under the Taiwan game.

PROPOSITION 3: *Under Assumptions 1 and 2, for any given* (P, π), λ, $\mathcal{M}$, *and* $\mathcal{N}$, *let* $\tilde{\pi}$ *be the augmented priority score profile. Then, there exists a unique Nash equilibrium outcome of this game, which is Pareto efficient and equivalent to* $DA(P, \tilde{\pi})$.

PROOF:

By our construction of $\tilde{\pi}$, when each sophisticated student i ranks $DA(P, \tilde{\pi})(i)$ as top choice and each sincere student j states P_j, TM^λ selects $DA(P, \tilde{\pi})$ and no sophisticated student profitably deviates. Hence, $DA(P, \tilde{\pi})$ is a Nash equilibrium outcome, and by Proposition 2, it is Pareto efficient.

Since under $(P, \tilde{\pi})$ there exists a unique stable matching (by Proposition 2), we will prove that there cannot be a Nash equilibrium outcome that is not stable under $(P, \tilde{\pi})$.

On the contrary, let Q be a Nash equilibrium profile, and the outcome of TM^λ under this strategy profile is μ; i.e., $TM^\lambda(Q, \pi) = \mu$, and μ is not stable under $(P, \tilde{\pi})$. Note that, $TM^\lambda(Q, \pi) = DA(Q, \hat{\pi}^\lambda)$, where $\hat{\pi}^\lambda$ is implied by (Q, π) and deduction rule λ.

If matching μ is individually irrational under $(P, \tilde{\pi})$, then there exists a student i who is assigned to an unacceptable school, i.e., $\varnothing \, P_i \, \mu(i)$. Since $Q_j = P_j$ for each $j \in \mathcal{N}$ and TM^{λ} is individually rational, i cannot be a sincere student. Then, individual rationality of TM^{λ} implies that ranking $\varnothing$ as top choice is a profitable deviation for i.

Suppose μ is wasteful or not fair under $(P, \tilde{\pi})$. Then, there exists a student-school pair (i,s) such that $s\ P_i\ \mu(i)$ and either $|\mu(s)| < q_s$ or $\tilde{\pi}_s(i) > \tilde{\pi}_s(j)$ for some $j \in \mu(s)$. Under both cases, we consider the sequential version of DA under $(Q, \hat{\pi}^{\lambda})$ such that i applies last. We first suppose the former case holds. Since $Q_j = P_j$ for each $j \in \mathcal{N}$ and TM^{λ} is nonwasteful, i cannot be a sincere student. Then, the population monotonicity of DA implies that i can profitably deviate by ranking school s as top choice. Now, we consider the latter case. By our construction, if $i \in \mathcal{N}$ and $\tilde{\pi}_s(i) > \tilde{\pi}_s(j)$, then $\hat{\pi}_s^{\lambda}(i) > \hat{\pi}_s^{\lambda}(j)$. Hence, i cannot be sincere. If i is sophisticated, then the fact that DA is population monotonic, the construction of $\tilde{\pi}$, and the proof of Lemma 2 imply that i can profitably deviate by ranking school s first. Hence, Q cannot be a Nash equilibrium. ∎

This result shows that the Taiwan mechanism favors sophisticated students over sincere students since, in equilibrium, deduction only applies to sincere students.

B. *Becoming Sophisticated*

What happens to a sincere student who learns the rules of the mechanism? In our example, a student who becomes sophisticated becomes (weakly) better off.

Example 1 (continued)**:** If i_2 and i_4 are sophisticated, under TM^{λ^1} the unique equilibrium outcome is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & d & c \end{pmatrix}.$$

If i_3 becomes sophisticated, then the augmented priority orders associated with $\tilde{\pi}$ are as follows:

a:	i_1	i_2	i_3	i_4
b:	i_1	i_2	i_3	i_4
c:	i_1	i_2	i_3	i_4
d:	i_2	i_1	i_3	i_4

The unique equilibrium outcome when i_2, i_3, and i_4 are sophisticated is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

Hence, by becoming sophisticated, i_3 is better off.

This example illustrates a more general phenomenon summarized by our next proposition.

PROPOSITION 4: *Under Assumptions 1 and 2, for any given* (P,π), λ, $\mathcal{M}$, *and* $\mathcal{N}$, *if a sincere student* $i \in \mathcal{N}$ *becomes sophisticated, then under the equilibrium outcome of* TM^{λ}, *i becomes (weakly) better off.*

PROOF:

Suppose that $\tilde{\pi}^1$ is the augmented priority score profile when i is sincere, and $\tilde{\pi}^2$ is the augmented priority score profile when i becomes sophisticated. By definition, for all $s \in S$, $\tilde{\pi}^1_s(j) = \tilde{\pi}^2_s(j)$ for all $j \neq i$ and $\tilde{\pi}^1_s(i) \leq \tilde{\pi}^2_s(i)$. That is, i is the unique improved student under the associated augmented priority orders when she becomes sophisticated. By Proposition 3, under both cases the unique equilibrium outcome is equivalent to $DA(P,\tilde{\pi}^1)$ and $DA(P,\tilde{\pi}^2)$, respectively. Since the DA respects improvement in priorities (see Balinski and Sönmez 1999),

$$DA\big(P,\tilde{\pi}^2\big)(i)\ R_i\ DA\big(P,\tilde{\pi}^1\big)(i). \blacksquare$$

When one sincere student becomes sophisticated, she obtains a weakly better assignment. Does this imply that other sophisticated students obtain weakly worse assignments? The answer turns out to be negative, as the following example illustrates.

Example 2: There are four schools, $S = \{a,b,c,d\}$, each with one seat, and four students, $I = \{i_1,i_2,i_3,i_4\}$. The student priority scores for each school $s \in S$ are $\pi_s(i_1) = 100$, $\pi_s(i_2) = 50$, $\pi_s(i_3) = 11$, and $\pi_s(i_4) = 0$. The preferences of the students are as follows:

P_{i_1}:	a	b	c	d	$\varnothing$
P_{i_2}:	a	b	d	c	$\varnothing$
P_{i_3}:	b	c	d	a	$\varnothing$
P_{i_4}:	d	c	a	b	$\varnothing$

Consider the deduction rule: $\lambda^1 = (0,41,45,51,51)$. Suppose initially that only i_4 is sophisticated. Then, under TM^{λ^1}, the unique equilibrium outcome is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & d & b & c \end{pmatrix}.$$

Now consider the case where i_2 becomes sophisticated. Then, under TM^{λ^1} when i_2 and i_4 are sophisticated, the unique equilibrium outcome is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

Therefore, student i_4 is better off after i_2 becomes sophisticated.

C. *Changing Mechanisms*

Since the Taiwan mechanism is manipulable, it is natural to compare it to a nonmanipulable mechanism. The most natural alternative is the serial dictatorship (SD) mechanism, a strategy-proof and efficient mechanism. In the setting where each school uses the same score, the DA produces the same outcome as an SD.

The first question we examine is whether sophisticated students prefer the Taiwan mechanism over an SD. Policy discussions in the city of Boston about the mechanism centered on the fact some families were exploiting their strategic knowledge to the detriment of families who do not have similar knowledge. Does the same argument apply in Taiwan?

The answer is no: a sophisticated student may in fact prefer the SD (and, therefore, the DA) over the Taiwan mechanism. In Example 2, under the SD, we obtain matching

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

Under the Nash equilibrium where only student i_4 is sophisticated, we obtain matching

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & d & b & c \end{pmatrix}.$$

Since i_4 prefers d to c, she is worse off under the unique equilibrium outcome of the Taiwan mechanism.

We also do not have a clear welfare comparison for sincere students. Example 1 illustrates that a sincere student need not be better off under this strategy-proof alternative.

Example 1 (continued)**:** When all students are sincere, student i_3 is assigned to a more preferred school b under TM^{λ^1} rather than school c under the SD.

So why has the Taiwan mechanism persisted for so many years in the face of massive condemnation and street protests? One reason is that not everyone, even sincere students, would be better off under the strategy-proof alternative. Indeed, this sentiment was expressed by Education Minister Chiang Wei-ling (Wei 2014) when he said no new policy would be carried out unless it would “benefit all students.”

D. *Simulations*

In Section IIC, we showed that there is no clear formal comparison between the serial dictatorship mechanism and the Taiwan mechanism in terms of student welfare. In particular, replacing a Taiwan mechanism with a serial dictatorship mechanism may benefit some sophisticated students and hurt some sincere students. In this section, to see the net effect of a possible shift from a Taiwan mechanism to a serial dictatorship mechanism, we consider simulating different scenarios. In our simulations, we consider environments with 50 schools, each with 300 available seats, and deduction points that increase by 2, 5, and 10 points. That is, if deduction points increase by $d \in \{2, 5, 10\}$, then the deduction rule is $\lambda = (0, d, 2d, \ldots, 50d)$. The number of students is equal to the total capacities of the schools, i.e., there are 15,000 students. These numbers are similar to those in Table 1, where the average number of applicants and schools across the 15 districts are 15,175, and 57, respectively.

To construct the preference of each student i, we calculate her utility from being assigned to each school s as follows:

$$U_{i,s} = \alpha X_s + (1 - \alpha) Y_{is},$$

where $X_s \in (0,1)$ and $Y_{is} \in (0,1)$ represents all students' common taste and student i's individual taste for school s, respectively. Both X_s and Y_{is} are i.i.d. standard uniformly distributed random variables. The level of correlation in the preferences of students is captured by $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$; i.e., as α increases, preferences become more correlated. The utility values of students are used to construct the ordinal preferences of students over the schools. For each student i, we randomly select a test score $\pi(i) \in (0, 100)$, which is an i.i.d. uniformly distributed random variable.

In simulations, we calculate the outcome of the serial dictatorship mechanism and the unique equilibrium outcome of the Taiwan mechanism, which is equivalent to the outcome of the DA under an augmented priority score profile, when the percentages of sophisticated students are taken as 0 percent, 25 percent, 50 percent, and 75 percent.[8]

Under each scenario, we calculate the outcome of the serial dictatorship mechanism and the equilibrium outcome of the Taiwan mechanism 100 times by using different draws for X, Y, and π. We present the fraction of sincere and sophisticated students preferring the outcome of the serial dictatorship mechanism over the equilibrium outcome of the Taiwan mechanism and vice versa in Figures 1 and 2, respectively. In online Appendix B, Figure 3 shows simulation results for all students.[9] We provide simulation results for 5,000 and 10,000 students in online Appendix B.

Each graph illustrates the fraction of students preferring one mechanism over the other and the fraction of students who have no preference, given different percentages of sophisticated students.[10] The horizontal axis refers to changing the combinations of deduction points and α.

Figure 1 shows that the serial dictatorship mechanism is preferred by a larger fraction of sincere students when deduction points increase, preferences become more correlated, and there are more sophisticated students. When the level of preference correlation increases from $\alpha = 0.25$ to $\alpha = 0.75$, a larger fraction of sincere

[8] When all students are sophisticated, the equilibrium outcome of the Taiwan mechanism and the outcome of the serial dictator mechanism coincide. Hence, we do not consider the scenario where the percentage of sophisticated students is 100 percent.

[9] Notice that online Appendix B Figure 3 shows the weighted averages of Figures 1 and 2 for the corresponding percentages of the sophisticated students.

[10] In Figure 2, we cannot provide simulation results for sophisticated students when the percentage of sophisticated students is 0 percent.

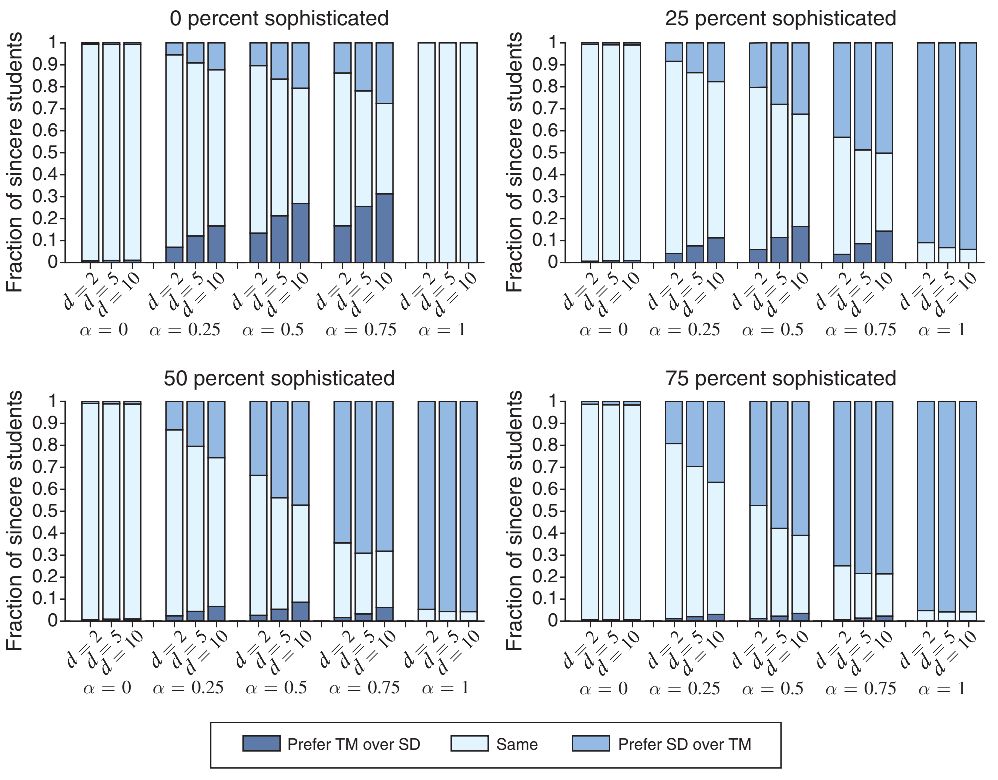

Figure 1. Simulation Results for Sincere Students (15,000 Students)

students prefer the outcome of the serial dictatorship mechanism over the equilibrium outcome of the Taiwan mechanism. The fraction of sincere students who prefer the serial dictatorship mechanism is largest in the extreme case when $\alpha = 1$, whenever the fraction of sophisticated students is nonzero. As the proportion of sophisticated students in the simulation increases, a larger fraction of sincere students prefer the serial dictatorship mechanism, for all values of α. However, with the exception of perfectly correlated preferences, sincere students do not all prefer the serial dictatorship mechanism. When $\alpha \neq 1$, there are always some sincere students who prefer the Taiwan mechanism over the serial dictatorship mechanism.

Figure 2 shows that in nearly all cases, no sophisticated students prefer the outcome of the serial dictatorship mechanism over the equilibrium outcome of the Taiwan mechanism. As the levels of preference correlation and deduction points increase, the fraction of sophisticated students preferring the equilibrium outcome of the Taiwan mechanism over the outcome of the serial dictatorship mechanism increases. As the percentage of sophisticated students increases, more sophisticated students become indifferent between two mechanisms.

Together, the two figures illustrate how the conflict between sincere and sophisticated students varies with the features of the economy. More sincere students prefer the serial dictatorship mechanism when deduction points are largest and preferences

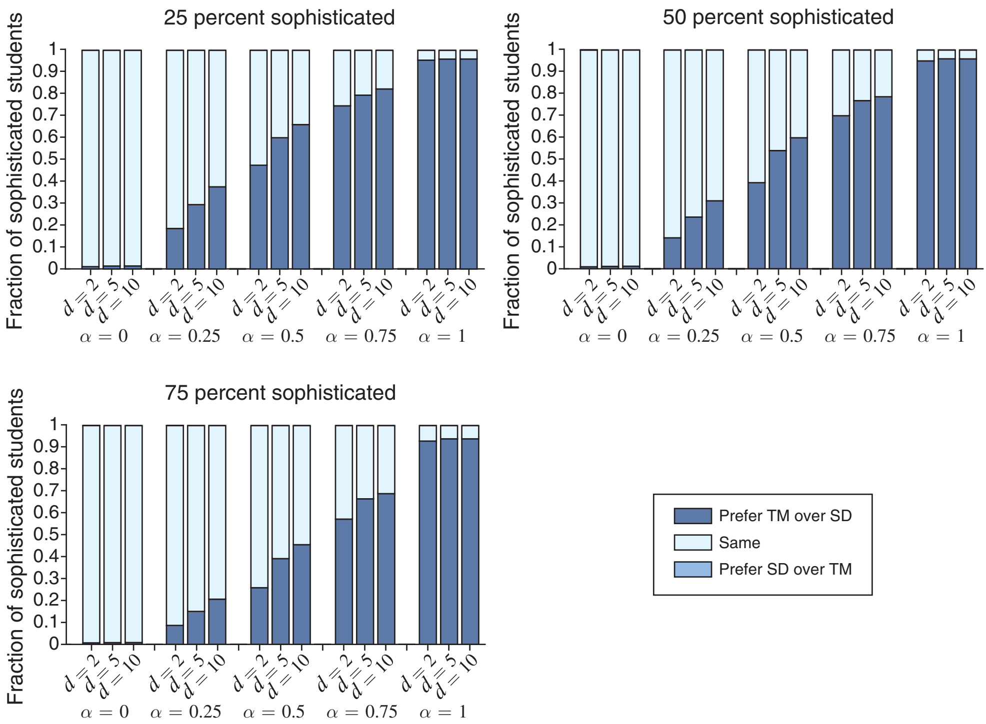

Figure 2. Simulation Results for Sophisticated Students (15,000 Students)

are more correlated. More sophisticated students prefer the Taiwan mechanism when deduction points are largest and preferences are more correlated. While sophisticated students are nearly unanimous in their preferences, sincere students are not. These simulations show quantitatively that not everyone, even sincere students, would be better off under the serial dictatorship mechanism.

III. Conclusion

A new Taiwanese school assignment mechanism has generated widespread turmoil and protests. This paper reports on the incentive properties of this mechanism and characterizes the equilibrium of the induced preference revelation game. Our results show that any mechanism using (nonzero) deduction is manipulable, and that the scope for manipulation increases with the size of the deductions. With sincere and sophisticated players, the Taiwan mechanism has a unique equilibrium, which can be characterized in terms of a stable matching of an alternative economy, where deduction applies to sincere students.

Our analysis provides a rationale for the reluctance of Taiwanese authorities to move to a strategy-proof alternative, illustrating a broader dynamic seen with manipulable mechanisms used in school choice and elsewhere. Boston Public Schools abandoned their mechanism in 2005, citing the desire to level the playing field between participants who understand the rules of the mechanism and those who do not. Pathak and Sönmez (2008) formalize the sense in which sophisticated players may prefer the manipulable mechanism. Under the Taiwanese mechanism, no particular group, sophisticated or sincere, would have common interests in the choice of mechanisms. Simulation results show that although sophisticated students would mostly prefer the Taiwan mechanism over a serial dictatorship mechanism in equilibrium, there are some sophisticated students who would prefer replacing the Taiwan mechanism with a serial dictatorship mechanism. This fact illustrates the broader theme that changes in market designs rarely involve Pareto improvements for even well-identified sets of participants, and this may stand in the way of reforms.

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